9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

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4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3 + 2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{6x}\right)(3 + 2x)^6$$

(3)

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1. The first three terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^{16}$ are

$$1, \quad -4x \quad \text{and} \quad px^2$$

where k and p are constants.

(a) Find, in simplest form,

(i) the value of k

(ii) the value of p

(3)

$$g(x) = \left(2 + \frac{16}{x}\right)(1 + kx)^{16}$$

Using the value of k found in part (a),

(b) find the term in x^2 in the expansion of $g(x)$.

(3)

DO NOT WRITE IN THIS AREA

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

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4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

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1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)

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2. A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where k is a positive constant.

(a) Find, in terms of k, an expression for u_3

(2)

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of k.

(3)

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7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

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8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

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