5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N,

(b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

(c) Hence, state the value of N.

(1)

1. The sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^{4}(u_r + 2r)$

(3)

2. The table shows corresponding values of x and y for a continuous curve with equation $y = f(x)$ between $x = -4$ and $x = 5$, where a is a constant.

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} f(x)\,dx$$

Given that the value of this approximation is 19.3

(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} \left(2f(x) - 3\right)dx$$

(2)

6.

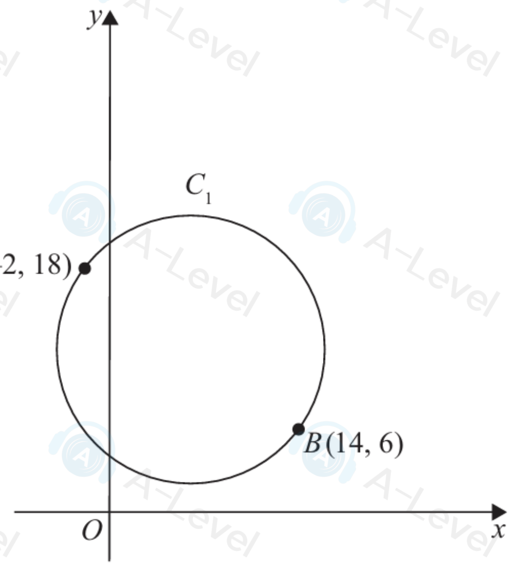

Figure 2

The points $A(-2, 18)$ and $B(14, 6)$ lie on a circle C_1 as shown in Figure 2.

Given that AB is a diameter of the circle C_1

(a) find an equation for C_1 making your method clear.

(5)

A circle C_2 has its centre at the origin.

Given that circles C_1 and C_2 touch,

(b) find possible equations for C_2

(4)

10:

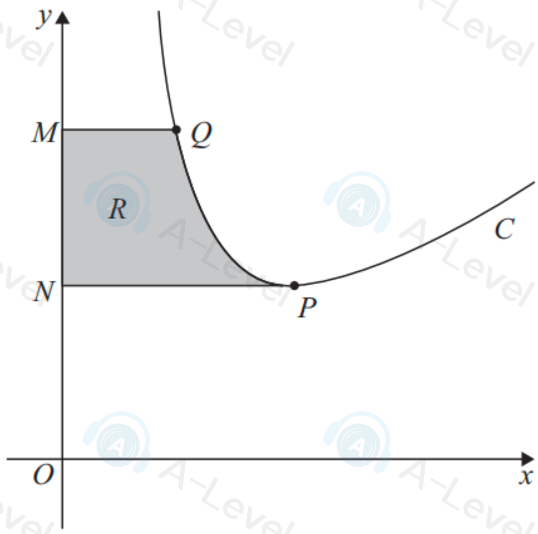

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with equation

$$y = 2x + \frac{64}{x^2} - 3 \qquad x > 0$$

The point P, shown in Figure 3, is the stationary point on C.

(a) Show, using calculus, that the x coordinate of P is 4

(4)

The point Q lies on C and has x coordinate 2

The line segments MQ and NP, shown in Figure 3, are parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, the y-axis and line segments MQ and NP.

(b) Use algebraic integration to find the exact area of R.

You must make your method clear.

(5)

2: A geometric series has first term a and common ratio r.

Given that

- the third term in the series is 64
- the sixth term in the series is –8

(a) show that $r = -\frac{1}{2}$

(2)

(b) find the sum to infinity of the series.

(4)

11. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where a and b are constants.

(a) Find, in terms of a and b,

(i) u_2

(ii) u_3

(2)

Given

- $\sum_{n=1}^{3} u_n = 153$
- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of u_2

(3)

5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta + \frac{2}{\tan\theta}\right) \equiv \sin\theta + \frac{2}{\sin\theta} \qquad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x + \frac{2}{\tan x}\right) = 4\sin x - 5$$

giving your answers to 3 significant figures.

(4)

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A sequence $u_1, u_2, u_3,$ is defined by

$$u_1 = 1$$
$$u_{n+1} = k - \frac{8}{u_n} \qquad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for u_2 and u_3 in terms of k.

(2)

Given that $u_3 = 6$

(b) find the possible values of k.

(3)

(Total for Question 1 is 5 marks)

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2(4 - x) = 3 + \log_2\left(\frac{x + 11}{2}\right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \qquad \text{and} \qquad y = 6 \times 3^x$$

meet at the point P.

Find the exact coordinates of P, writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Given that

$$\log_a x + \log_a 3 = \log_a 27 - 1 \qquad \text{where } a \text{ is a positive constant}$$

find, in its simplest form, an expression for x in terms of a.

(4)

(ii) Solve the equation

$$(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$$

showing each step of your working.

(4)

(Total for Question 5 is 8 marks)

8. **Solutions relying on calculator technology are not acceptable in this question.**

(i)

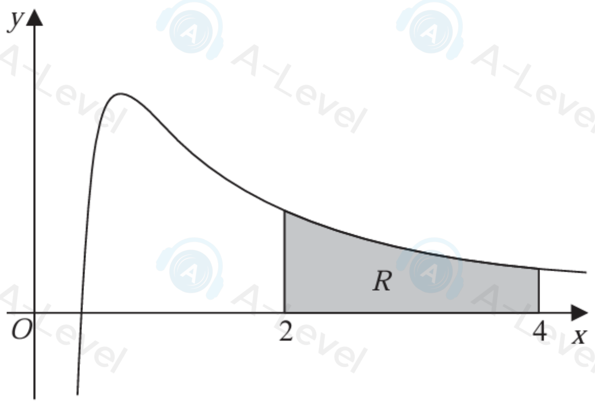

Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) \mathrm{d}x = 55$$

(4)