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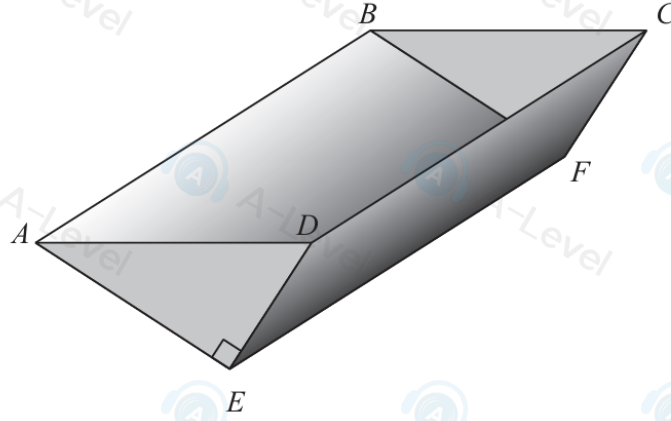


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is 12 m^3

(a) show that the area of metal used to make the container, $S \text{ m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S , giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

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5:

x	-2	-0.5	1	2.5	4	5.5	7
y	12	4.243	1.5	0.530	0.188	0.066	0.023

The table above shows corresponding values of x and y for

$$y = 3\left(\frac{1}{2}\right)^x$$

The values of y are given to 3 decimal places as appropriate.

- (a) Using the trapezium rule with all the values of y in the given table, obtain an estimate for

$$\int_{-2}^7 3\left(\frac{1}{2}\right)^x dx$$

giving the answer to one decimal place.

(3)

Using the answer to part (a) and making your method clear, estimate

(b) (i) $\int_{-2}^7 3\left(\frac{1}{2}\right)^{x+2} dx$

(ii) $\int_{-2}^7 (2^{-x} + 2x) dx$

(3)

8. (i) Use a counter example to show that the following statement is **false**

$$“n^2 + 3n + 1 \text{ is prime for all } n \in \mathbb{N}”$$

(2)

- (ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

$$“n^2 - 2 \text{ is not a multiple of } 4”$$

(4)

6. (i) The circle C_1 has equation

$$x^2 + y^2 + 10x - 12y = k \quad \text{where } k \text{ is a constant}$$

(a) Find the coordinates of the centre of C_1

(2)

(b) State the possible range in values for k .

(2)

(ii) The point $P(p, 0)$, the point $Q(-2, 10)$ and the point $R(8, -14)$ lie on a different circle, C_2

Given that

- p is a positive constant
- QR is a diameter of C_2

find the exact value of p .

(4)

3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geq 2x - 9$$

(3)

(ii) Show that the following statement is untrue.

$$2^n + 1 \text{ is a prime number for all values of } n, n \in \mathbb{N}$$

(1)

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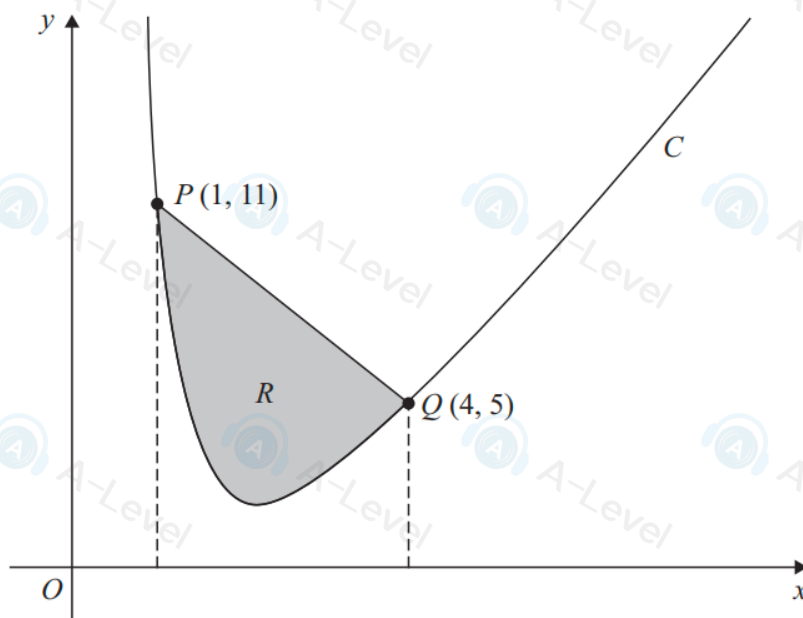


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with equation

$$y = 3x + \frac{16}{x^2} - 8 \quad x > 0$$

The points $P(1, 11)$ and $Q(4, 5)$ lie on C and are shown in Figure 1.

The region R , shown shaded in Figure 1, is bounded by C and line segment PQ .

Use algebraic integration to find the area of R .

(5)

8.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

- (i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5 \sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

- (ii) Solve, for $0 < \theta < 360^\circ$, the equation

$$2 \tan \theta \sin \theta = 5 + \cos \theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

1. Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\left(2 + \frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)