3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) a_1

(ii) a_2

(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$$

You must make your method clear.

(4)

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

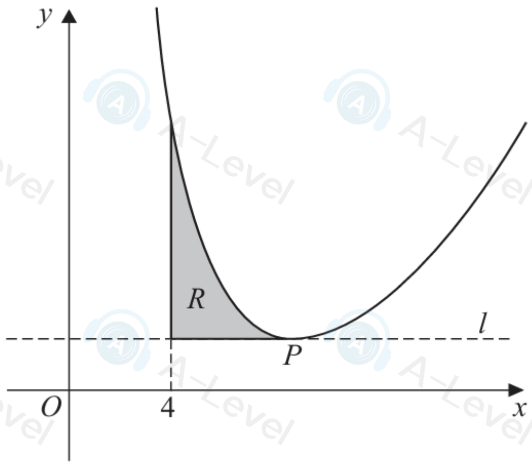

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9 **(4)**

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R. **(5)**

DO NOT WRITE IN THIS AREA

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 360°$, the equation

$$4 \tan \theta + 5 \sin \theta = 0$$

giving any non-exact answers to one decimal place. **(5)**

(ii) Solve, for $0 < x < \pi$, the equation

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\cos x}$$

giving the answers, in radians, to 3 significant figures. **(4)**

DO NOT WRITE IN THIS AREA

5 A curve has equation $4e^{2x}y + y^2 = 21$.

Find the gradient of the curve at the point $(0, -7)$. [5]

1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

1. A continuous curve has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below.

x	0.5	1.75	3	4.25	5.5
y	3.479	6.101	7.448	6.823	5.182

Using the trapezium rule with all the values of y in the given table,

(a) find an estimate for

$$\int_{0.5}^{5.5} f(x)\,dx$$

giving your answer to one decimal place.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

$$\int_{0.5}^{5.5} (f(x) + 4x)\,dx$$

(2)

2: A geometric series has first term a and common ratio r.

Given that

- the third term in the series is 64
- the sixth term in the series is –8

(a) show that $r = -\frac{1}{2}$ **(2)**

(b) find the sum to infinity of the series.

(4)

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a polynomial in x.

Given

$$\mathrm{f}'(x) = 2(3x - 2)(x + 5)$$

(a) deduce the range of values of x for which y is decreasing.

(2)

Given further that C cuts the x-axis at $\frac{7}{2}$

(b) state a factor of $\mathrm{f}(x)$, giving the answer in the form $(Ax + B)$ where A and B are integers.

(1)

(c) Hence find an expression for $\mathrm{f}(x)$ in a fully factorised form.

(6)

blank

1. $$\mathrm{f}(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $\mathrm{f}(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $\mathrm{f}(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

3.

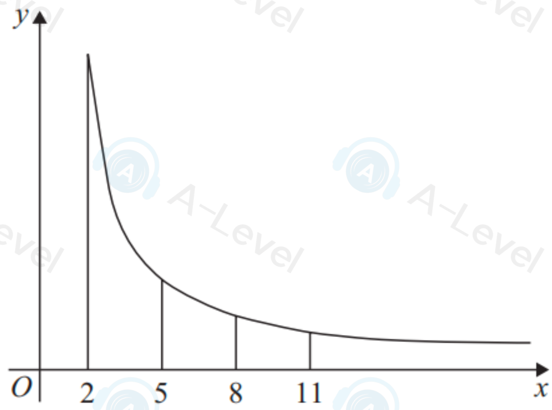

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \dfrac{12}{\sqrt{x^2 - 2}}$ $\quad x \geqslant 2$

The table below gives values of y rounded to 3 decimal places.

x	2	5	8	11
y	8.485	2.502	1.524	1.100

(a) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for

$$\int_2^{11} \frac{12}{\sqrt{x^2 - 2}}\,dx$$

(3)

(b) Use your answer to part (a) to estimate a value for

$$\int_2^{11} \left(1 + \frac{6}{\sqrt{x^2 - 2}}\right) dx$$

(3)

(Total for Question 3 is 6 marks)

4. $$f(x) = (x - 2)(2x^2 + 5x + k) + 21$$

where k is a constant.

(a) State the remainder when $f(x)$ is divided by $(x - 2)$

(1)

Given that $(2x - 1)$ is a factor of $f(x)$

(b) show that $k = 11$

(2)

(c) Hence

(i) fully factorise $f(x)$,

(ii) find the number of real solutions of the equation

$$f(x) = 0$$

giving a reason for your answer.

(5)