

2. The first 4 terms, in ascending powers of  $x$ , in the binomial expansion of

$$(1 + px)^{10}$$

are

$$1 + 15x + qx^2 + rx^3$$

where  $p, q$  and  $r$  are constants.

Find the value of  $p$ , the value of  $q$  and the value of  $r$ .

(6)

9. Given that

$$3 \log_2(t + 4) - 2 \log_2(t - 2) = 7$$

- (a) verify that  $t = 4$  is a solution of the above equation,

(2)

- (b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

- (c) Hence, using algebra and showing your working, solve

$$3 \log_2(t + 4) - 2 \log_2(t - 2) = 7$$

giving each answer in simplest form.

*(Solutions based entirely on calculator technology are not acceptable.)*

(4)

5. (a) Find, in terms of  $a$ , the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$(2 + ax)^6$$

where  $a$  is a non-zero constant. Give each term in simplest form.

(3)

$$f(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

Given that the constant term in the expansion of  $f(x)$  is 576

- (b) find the value of  $a$ .

(4)

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7: **In this question you must show all stages of your working.**  
**Solutions relying entirely on calculator technology are not acceptable.**

The binomial expansion, in ascending powers of  $x$ , of  $(1 + kx)^n$  is

$$1 - 24x + 270x^2 + px^3 + \dots$$

where  $k$  and  $p$  are constants and  $n$  is an integer greater than 1

(a) Use this information to set up and solve two simultaneous equations to find the value of  $k$  and the value of  $n$ . (6)

(b) Hence find the value of  $p$ . (2)

6. (a) Given that

$$2 \log_4(x + 3) + \log_4 x = \log_4(4x + 2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$
(4)

(b) Given also that  $-1$  is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation. (3)

(ii) Hence solve

$$2 \log_4(x + 3) + \log_4 x = \log_4(4x + 2) + \frac{1}{2}$$
(1)

2. The first 4 terms, in ascending powers of  $x$ , in the binomial expansion of

$$(1 + px)^{10}$$

are

$$1 + 15x + qx^2 + rx^3$$

where  $p$ ,  $q$  and  $r$  are constants.

Find the value of  $p$ , the value of  $q$  and the value of  $r$ . (6)

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$
(5)

1 Given that

$$\ln(2x + 1) - \ln(x - 3) = 2,$$

find  $x$  in terms of  $e$ .

[4]

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- $N$  grams is the amount of carbon-14 **currently** present in the item
- $k$  grams was the **initial** amount of carbon-14 present in the item
- $t$  is the number of years since the item was made
- $\lambda$  is a constant, with  $0 < \lambda < 1$

(a) Sketch the graph of  $N$  against  $t$  for  $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant  $\lambda$  is 0.999878 to 6 decimal places.

(2)

Given that Item  $A$

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item  $A$ .

(2)

Item  $B$  is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item  $B$  to the nearest 100 years.

(3)

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4. (a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$(2 + px)^6$$

where  $p$  is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of  $x^2$  is  $-\frac{3}{4}$

- (b) find the possible values of  $p$ .

(4)

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3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form  $x = \log_7 a$  where  $a$  is a rational number in its simplest form.

(3)

- (ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

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4. The binomial expansion, in ascending powers of  $x$ , of

$$(3 + px)^5$$

where  $p$  is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \dots$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are constants.

- (a) Find the value of  $A$

(1)

Given that

- $B = 18D$
- $p < 0$

- (b) find

(i) the value of  $p$

(ii) the value of  $C$

(6)

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2. Find the coefficient of the term in  $x^7$  of the binomial expansion of

$$\left(\frac{3}{8} + 4x\right)^{12}$$

giving your answer in simplest form.

(3)