**9.** **In this question you must show detailed reasoning.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

**(3)**

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

**(4)**

DO NOT WRITE IN THIS AREA DO

blank

**1.** A curve $C$ has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the $x$ coordinates of the stationary points of $C$.

**(4)**

(b) Hence find the values of $x$ for which $y$ is increasing.

**(2)**

DO NOT WRIT

**6:** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

(i) Given that $\theta$ is measured in degrees and

- $\cos\theta = \frac{1}{\sqrt{5}}$
- $180° < \theta < 360°$

use trigonometric identities to find the exact value of

(a) $\sin\theta$

(b) $\tan\theta$

giving the answers as fully simplified surds where appropriate.

**(4)**

DO NOT WRITE IN THIS AREA

blank

**8.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\dfrac{dy}{dx}$

**(3)**

(b) Hence find the coordinates of the stationary points of the curve.

**(5)**

DO NOT WRITE IN THIS AREA

**8.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

**(5)**

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H°$C, inside the greenhouse, $t$ hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where $k$ is a constant.

**Use the equation of the model to answer parts (a) to (c).**

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for $k$, giving each answer to 2 decimal places.

**(4)**

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

**(1)**

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

**(2)**

DO NOT WRITE IN THIS AREA

**4.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The curve $C$ has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving each term in simplest form.

**(2)**

(b) Hence find the $x$ coordinate of the stationary point of $C$.

**(2)**

(c) (i) Find $\frac{d^2y}{dx^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of $C$, giving a reason for your answer.

**(2)**

(d) State the range of values of $x$ for which $y$ is decreasing.

**(1)**

**2.** **In this question you must show all stages of your working.**

**Solutions based entirely on calculator technology are not acceptable.**

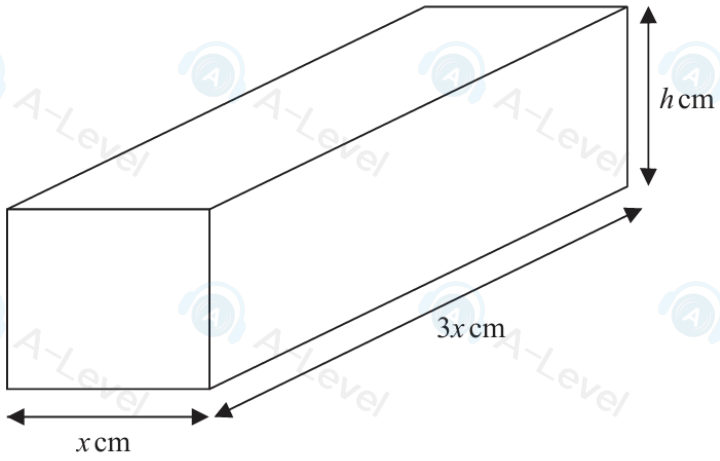


**Figure 2**

A brick is in the shape of a cuboid with width $x$ cm, length $3x$ cm and height $h$ cm, as shown in Figure 2.

The volume of the brick is $972\,\text{cm}^3$

(a) Show that the surface area of the brick, $S\,\text{cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

**(3)**

(b) Find $\dfrac{\mathrm{d}S}{\mathrm{d}x}$

**(1)**

(c) Hence find the value of $x$ for which $S$ is stationary.

**(2)**

(d) Find $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$ and hence show that the value of $x$ found in part (c) gives the minimum value of $S$.

**(2)**

(e) Hence find the minimum surface area of the brick.

**(1)**

7. **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$7\sin 2\theta = 5\cos 2\theta$$

giving your answers, in degrees, to one decimal place.

**(4)**

(ii) Solve, for $0 \leqslant x < 2\pi$, the equation

$$24\tan x = 5\cos x$$

giving your answers, in radians, to 3 decimal places.

**(5)**

**(Total for Question 7 is 9 marks)**

**9.** **In this question you must show detailed reasoning.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

**(6)**

(ii)

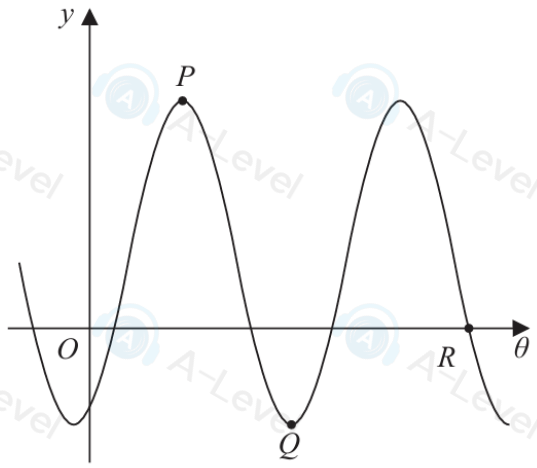


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where $A$ is a constant and $\theta$ is measured in radians.

The points $P$, $Q$ and $R$ lie on the curve and are shown in Figure 1.

Given that the $y$ coordinate of $P$ is 7

(a) state the value of $A$,

**(1)**

(b) find the exact coordinates of $Q$,

**(3)**

(c) find the value of $\theta$ at $R$, giving your answer to 3 significant figures.

**(4)**

**5** A curve has equation $4e^{2x}y + y^2 = 21$.

Find the gradient of the curve at the point $(0, -7)$. [5]

**10.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

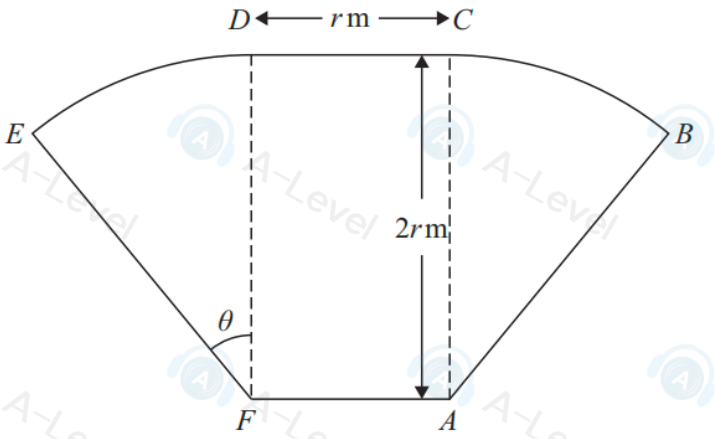


**Figure 2**

Figure 2 shows the plan view of the design for a stage at a trade fair.

The shape of the stage $ABCDEFA$, consists of a rectangle $ACDF$ joined to two congruent sectors of circles. $ABC$ is a sector of a circle centre $A$ and $FDE$ is a sector of a circle centre $F$.

Given that $AC = 2r$ metres, $CD = r$ metres, angle $DFE = \theta$ radians and the area of the stage is $30\,\text{m}^2$,

(a) show that the perimeter, $P$ metres, of the stage, is given by

$$P = 4r + \frac{30}{r}$$

**(4)**

(b) Use calculus to find the minimum value for $P$, giving your answer in the form $a\sqrt{b}$, where $a$ and $b$ are integers to be found.

**(4)**

(c) Justify that the value of $P$ found in part (b) is the minimum.

**(2)**

**(Total for Question 10 is 10 marks)**

3.

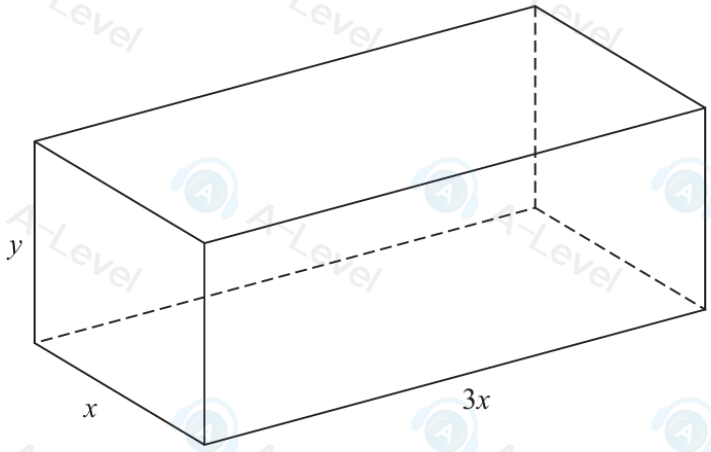


**Figure 1**

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by $x$ metres.

The height of the container is $y$ metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where $P$ and $Q$ are positive constants to be found.

**(4)**

(b) Use calculus to find the value of $x$ for which $A$ has a stationary value, giving your answer to 3 significant figures.

**(4)**

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of $x$ found in part (b) gives the minimum value of $A$.

**(2)**

4. **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

$$\mathrm{f}(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $\mathrm{f}(x)$ is divided by $(x - 2)$ the remainder is $R$ and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find $R$.

**(4)**

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $\mathrm{f}(x)$.

(ii) Hence prove, using algebra, that the equation $\mathrm{f}(x) = 0$ has only one real solution.

**(5)**

(c) Find the range of values of $x$ for which $\mathrm{f}(x)$ is decreasing.

**(3)**