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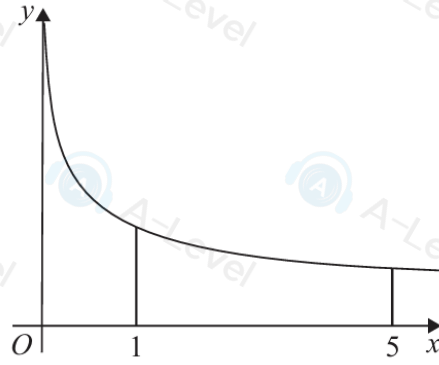


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \log_3(x+1) - \log_3 x$$

The point $P(a, 4)$ lies on the curve.

- (a) Find the exact value of the constant a .

(Solutions relying on calculator technology are not acceptable.)

(4)

- (b) Use the trapezium rule with 4 strips of equal width to estimate the value of

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

giving the answer in the form $\log_3 k$, where k is a constant to be found.

(4)

- (c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

(1)

10.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

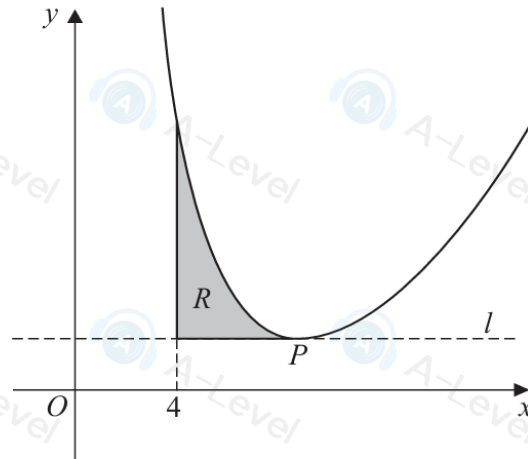


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \quad x > 0$$

The point P is the only stationary point on the curve.

- (a) Use calculus to show that the x coordinate of P is 9 (4)

The line l passes through the point P and is parallel to the x -axis.

The region R , shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

- (b) Use algebraic integration to find the exact area of R . (5)

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6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y -axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} (2^x - 2x) dx$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} (2^x + 2x) dx$

(ii) $\int_2^{3.5} (2^{x+1} - 4x) dx$

(3)

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2 \log_3(1 - x) + 3$$

(5)

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5 The polynomial $p(x)$ is defined by

$$p(x) = ax^4 + bx^3 + 13x^2 - 35x + 15,$$

where a and b are constants. It is given that $(2x - 1)$ and $(x - 3)$ are factors of $p(x)$.

(a) Find the values of a and b .

[4]

10.

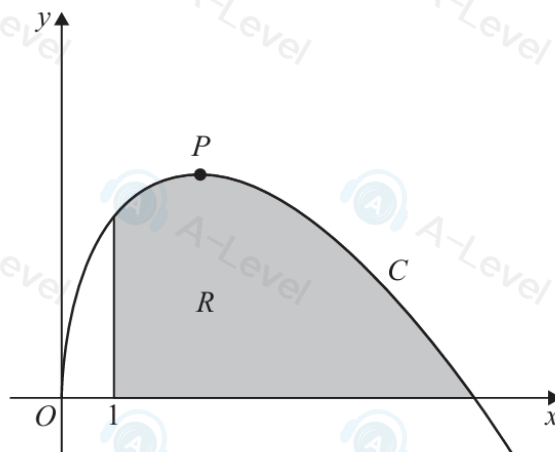


Figure 1

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \quad x > 0$$

The point P is the stationary point on C .

(a) Find, using calculus, the x coordinate of P .

(4)

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the x -axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R .

(5)

3 Find the coordinates of the stationary points of the curve with equation $y = \frac{8x}{2x+3} - 6x + 5$.

[5]

9.

**In this question you must show detailed reasoning.
Solutions relying on calculator technology are not acceptable.**

(i) Solve

$$2\log_3(4x + 5) - \log_3(x + 3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a + b)$$

(a) prove that $a = \frac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b , giving a reason for your answer.

(2)

4. (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

- (b) find the possible values of p .

(4)

7.

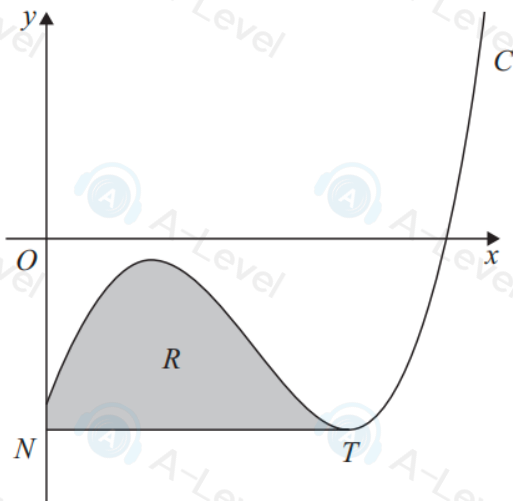


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \quad x \geq 0$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$ in simplest form.

(2)

The point T , shown in Figure 3, is a minimum stationary point on C .

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x -axis meets the y -axis at the point N .

The finite region R , shown shaded in Figure 3, is bounded by C , the y -axis and the line segment NT .

(c) Use algebraic integration to find the area of R , giving the answer to 3 significant figures.

(6)

1 Given that

$$\ln(2x + 1) - \ln(x - 3) = 2,$$

find x in terms of e .

[4]

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found.

(4)

- (ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1$$

(5)

4. (a) Find, in ascending powers of x , up to and including the term in x^3 , the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

- (b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

- (c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)