4 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - ax^2 - 15x + 18,$$

where a is a constant. It is given that $(x+2)$ is a factor of $p(x)$.

(a) Find the value of a. [2]

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(b) Hence factorise $p(x)$ completely. [3]

3.

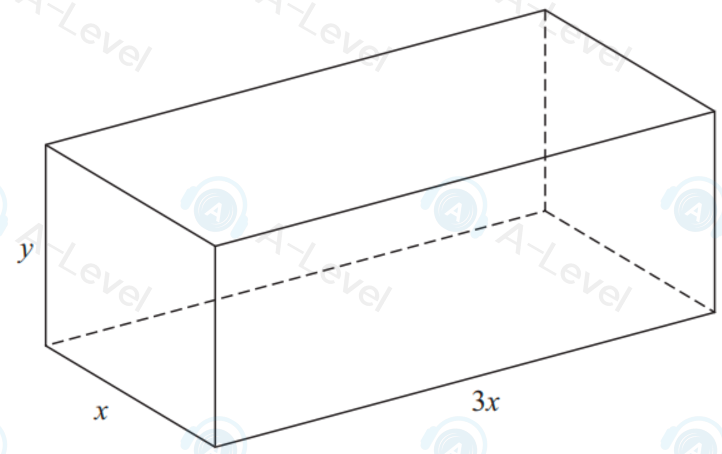

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Given that θ is measured in degrees and

- $\cos\theta = \frac{1}{\sqrt{5}}$
- $180° < \theta < 360°$

use trigonometric identities to find the exact value of

(a) $\sin\theta$

(b) $\tan\theta$

giving the answers as fully simplified surds where appropriate.

(4)

3. The circle C

- has centre $A(3, 5)$
- passes through the point $B(8, -7)$

(a) Find an equation for C.

(3)

The points M and N lie on C such that MN is a chord of C.

Given that MN

- lies above the x-axis
- is parallel to the x-axis
- has length $4\sqrt{22}$

(b) find an equation for the line passing through points M and N.

(3)

DO NOT WRITE IN THIS AREA

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

DO NOT WRITE IN THIS AREA

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

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5.

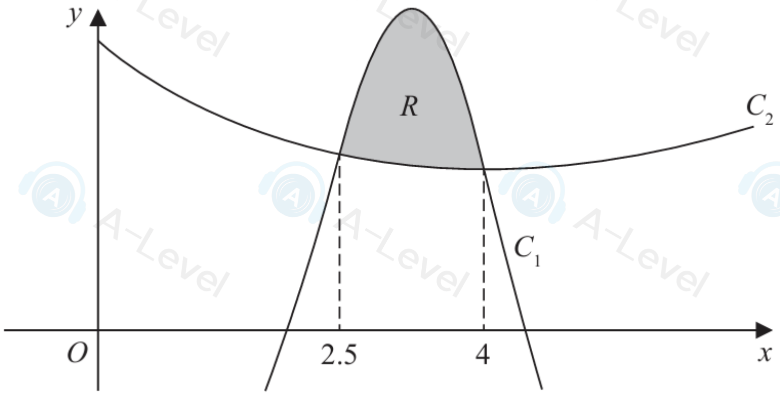

Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\int \left(x^{\frac{3}{2}} - 3x + 9 \right) dx$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

3: $$f(x) = (3x^2 - 4x - 5)(x - k) - 5$$

where k is a constant.

(a) Deduce the value of the remainder when $f(x)$ is divided by $(x - k)$

(1)

Given that the remainder when $f(x)$ is divided by $(x + 2)$ is 25

(b) show that the value of k is -4

(2)

(c) Hence find the quotient and remainder when $f(x)$ is divided by $(3x - 1)$

(4)

7.

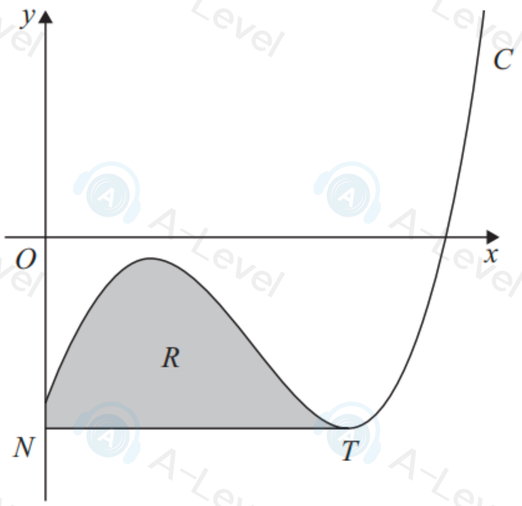

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

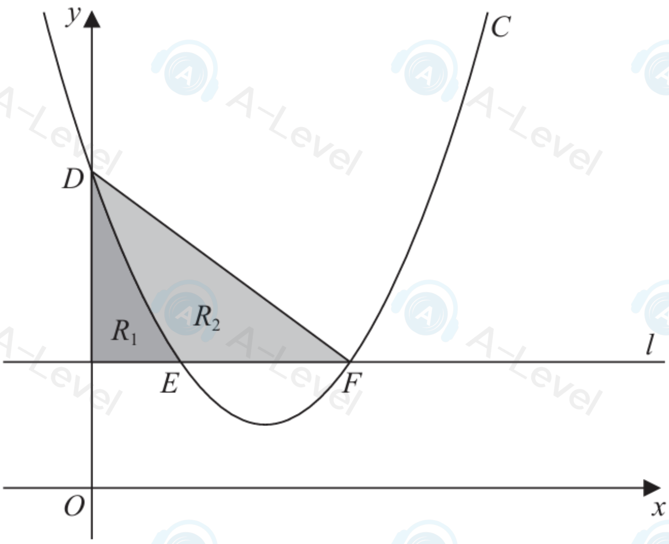

Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

9: The circle C

- has a centre which lies on the x-axis
- touches the y-axis
- passes through the point (5, 6)

(a) On Diagram 1, sketch a graph of C.

(1)

(b) Find an equation for C.

(4)

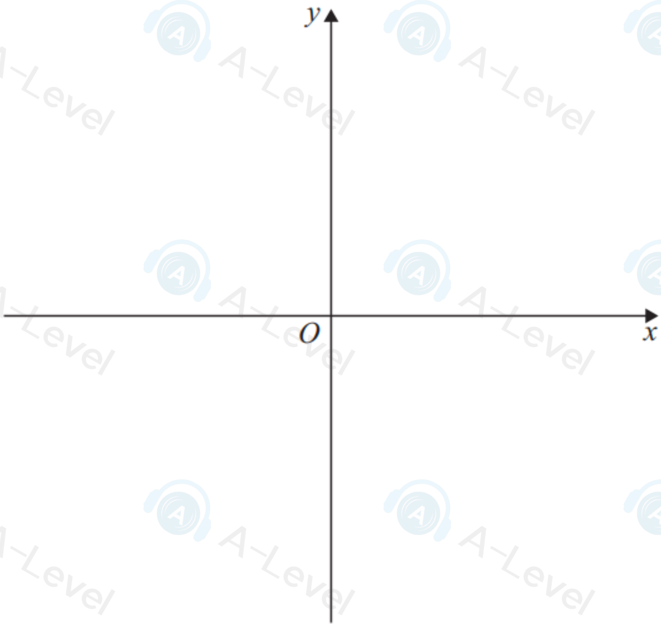

Diagram 1

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A sequence u_1, u_2, u_3, is defined by

$$u_1 = 1$$

$$u_{n+1} = k - \frac{8}{u_n} \qquad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for u_2 and u_3 in terms of k.

(2)

Given that $u_3 = 6$

(b) find the possible values of k.

(3)

(Total for Question 1 is 5 marks)

5:

x	-2	-0.5	1	2.5	4	5.5	7
y	12	4.243	1.5	0.530	0.188	0.066	0.023

The table above shows corresponding values of x and y for

$$y = 3\left(\frac{1}{2}\right)^x$$

The values of y are given to 3 decimal places as appropriate.

(a) Using the trapezium rule with all the values of y in the given table, obtain an estimate for

$$\int_{-2}^{7} 3\left(\frac{1}{2}\right)^x \mathrm{d}x$$

giving the answer to one decimal place.

(3)

Using the answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_{-2}^{7} 3\left(\frac{1}{2}\right)^{x+2} \mathrm{d}x$

(ii) $\displaystyle\int_{-2}^{7} \left(2^{-x} + 2x\right) \mathrm{d}x$

(3)