1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$" **(2)**

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4" **(4)**

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5 The polynomial $p(x)$ is defined by $p(x) = 9x^3 + 18x^2 + 5x + 4$.

(a) Find the quotient when $p(x)$ is divided by $(3x + 2)$, and show that the remainder is 6. [3]

1: Given that a, b and c are positive integers such that

- $c = a^2 - 1$
- $a + b + c = 20$

prove, by exhaustion, that the product abc is always a multiple of 6

You may use the table below to illustrate your answer. **(3)**

You may not need to use all rows of this table.

a	b	c	abc

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3. (i) Show that the following statement is **false**:

"$(n+1)^3 - n^3$ is prime for all $n \in \mathbb{N}$" **(2)**

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle. **(5)**

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10: **In this question you must show detailed reasoning.**

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

blank

9. (a) Prove that for all positive values of x and y,

$$\frac{x + y}{2} \geqslant \sqrt{xy}$$

(3)

(b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)

1 Solve the inequality $|2x - 5| > x$. [4]

blank

3. (i) Prove that for all single digit prime numbers, p,

$$p^3 + p \text{ is a multiple of } 10$$

(2)

(ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n + 1)^3 - n^3 \text{ is not a multiple of } 3$$

(3)