blank

2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

DO NOT WRITE IN THIS AREA

2. Find the first 3 terms in ascending powers of x of

$$\left(2 - \frac{x}{2}\right)^6$$

giving each term in its simplest form.

(4)

(Total for Question 2 is 4 marks)

6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

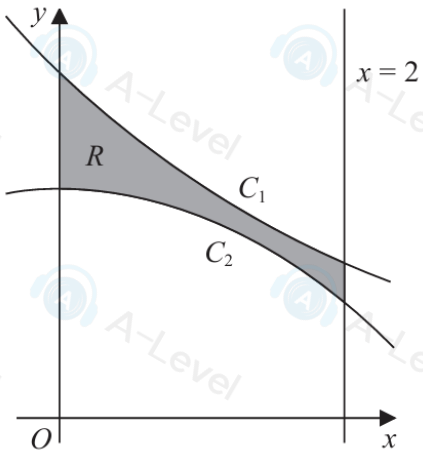

Figure 1

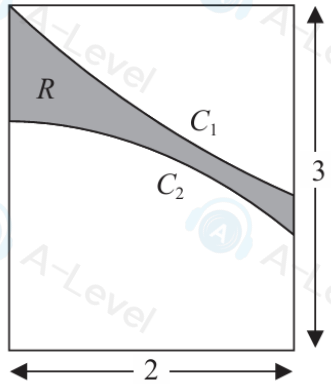

Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

blank

3. $f(x) = 6x^3 + 17x^2 + 4x - 12$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

DO NOT WRITE IN THIS AREA

blank

1. The table below shows corresponding values of x and y for $y = \log_2(2x)$

The values of y are given to 2 decimal places as appropriate.

x	2	5	8	11	14
y	2	3.32	4	4.46	4.81

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for $\int_2^{14} \log_2(2x)\,dx$, giving your answer to one decimal place.

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_2^{14} \frac{\log_2(4x^2)}{5}\,dx$

(ii) $\int_2^{14} \log_2\left(\frac{2}{x}\right)dx$

(4)

DO NOT WRITE IN THIS AREA

DO

blank

6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

DO NOT WRITE IN THIS AREA

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The binomial expansion, in ascending powers of x, of $(1 + kx)^n$ is

$$1 - 24x + 270x^2 + px^3 + \ldots$$

where k and p are constants and n is an integer greater than 1

(a) Use this information to set up and solve two simultaneous equations to find the value of k and the value of n.

(6)

(b) Hence find the value of p.

(2)

DO NOT WRITE IN THIS AREA

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

DO NOT WRITE IN THIS ARE

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

DO NOT WRITE IN THIS AREA

DO NC

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A car has five gears. Given that

- the maximum speed of the car in first gear is $22\,\text{km}\,\text{h}^{-1}$
- the maximum speed in each successive gear forms a geometric sequence
- the maximum speed of the car in fifth gear is $130\,\text{km}\,\text{h}^{-1}$

find the maximum speed of the car in second gear, giving your answer, in $\text{km}\,\text{h}^{-1}$, to one decimal place.

(4)

(ii) The first two terms of an arithmetic sequence are 208 and 207.2

Given that S_n is the sum to n terms,

(a) find the maximum value of S_n

(4)

(b) Hence or otherwise state the smallest value of N such that $S_N < 0$

(1)

(Total for Question 9 is 9 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

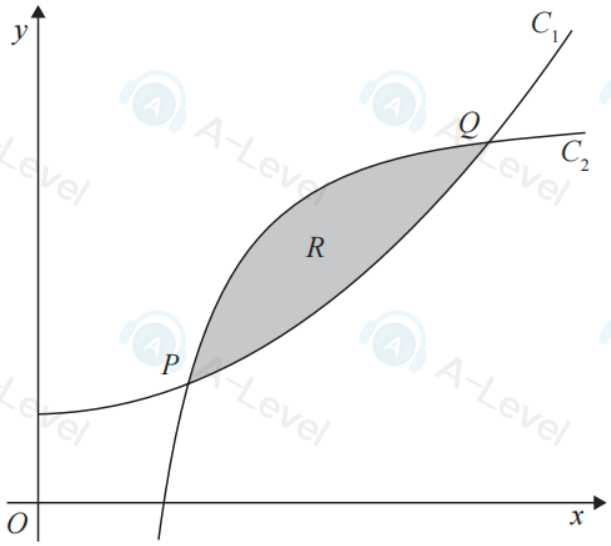

Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

9: The circle C

- has a centre which lies on the x-axis
- touches the y-axis
- passes through the point (5, 6)

(a) On Diagram 1, sketch a graph of C.

(1)

(b) Find an equation for C.

(4)

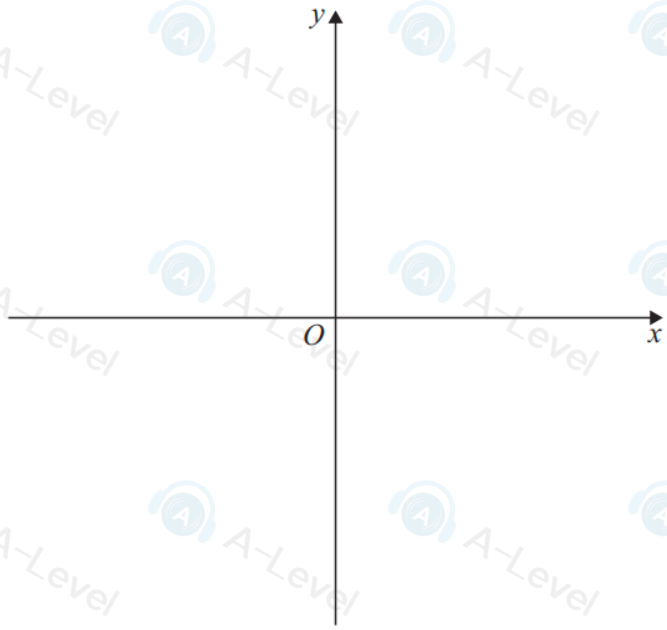

Diagram 1