5. $f(x) = 3x^3 + ax^2 - 10x + b$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

DO NOT WRITE IN THIS AREA

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4. The points P and Q have coordinates $(-11, 6)$ and $(-3, 12)$ respectively.

Given that PQ is a diameter of the circle C,

(a) (i) find the coordinates of the centre of C,

(ii) find the radius of C.

(4)

(b) Hence find an equation of C.

(2)

(c) Find an equation of the tangent to C at the point Q giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(3)

DO NOT WRITE IN THIS AREA

6. A circle C has equation $x^2 + y^2 - 6x - 14y + k = 0$ where k is constant.

(a) Find the coordinates of the centre of C.

(2)

(b) Find the radius of C when $k = -32$

(2)

(c) Find the range of values of k for which C lies completely within the first quadrant.

(4)

(Total for Question 6 is 8 marks)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

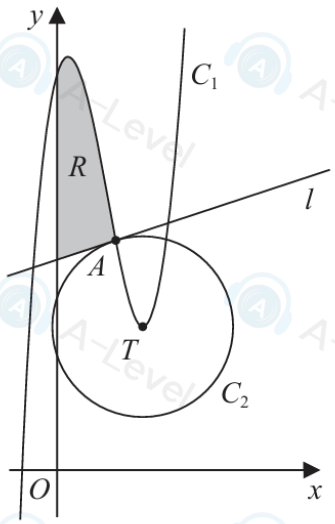

Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

blank

3. $f(x) = 6x^3 + 17x^2 + 4x - 12$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

DO NOT WRITE IN THIS AREA

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $f(x)$ is divided by $(x - 2)$ the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R.

(4)

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(ii) Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(5)

(c) Find the range of values of x for which $f(x)$ is decreasing.

(3)

DO NOT WRITE IN THIS AREA

10: **In this question you must show detailed reasoning.**

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

DO NOT WRITE IN THIS