11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

10. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$

$$2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$$

which is not divisible by 3

Complete this proof.

(4)

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$f(x) = 13 + 3x + (x + 2)(x + k)^2$ where k is a constant

Given that $(x + 3)$ is a factor of $f(x)$,

(a) (i) show that a possible value of k is 5

(ii) find the other possible value of k.

(3)

Given that $k = 5$

(b) (i) write $f(x)$ as the product of two algebraic factors

(ii) show that the equation $f(x) = 0$ has only one real solution.

(6)

(Total for Question 4 is 9 marks)

5. (i) Use algebra to prove that for all $x \geqslant 0$

$$3x + 1 \geqslant 2\sqrt{3x}$$

(3)

(ii) Show that the following statement is not true.

"The sum of three consecutive prime numbers is always a multiple of 5"

(1)

1. $f(x) = ax^3 + 3x^2 - 8x + 2$ where a is a constant

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a.

(3)

DO I

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $f(x)$ is divided by $(x - 2)$ the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R.

(4)

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(ii) Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(5)

(c) Find the range of values of x for which $f(x)$ is decreasing.

(3)

DO NOT WRITE IN THIS AREA

4: The function $f(x)$ is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that

$$b = 3a + 2$$

(2)

Given further that when $f(x)$ is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

(b) find the value of a and the value of b.

(4)

(c) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 2)$

(3)

DO NOT WRITE IN THIS AREA

DO

1: Given that a, b and c are positive integers such that

- $c = a^2 - 1$
- $a + b + c = 20$

prove, by exhaustion, that the product abc is always a multiple of 6

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c	abc

DO NOT WRITE IN THIS AREA DO NOT WRI

5. $$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51$$

(2)

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0$$

(2)

(c) Hence find the value of p and the value of q.

(3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x)$$

(2)

DO NOT WRITE IN THIS AREA DO NOT

6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

blank

DO NOT WRITE IN THIS AREA

9. (a) Prove that for all positive values of x and y,

$$\frac{x + y}{2} \geqslant \sqrt{xy}$$

(3)

(b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)

blank

DO NOT WRITE IN

3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geqslant 2x - 9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

blank

DO NOT WRITE IN T

4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $f(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $f(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $f(x)$.

(5)

blank

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