

4. (a) Find, in ascending powers of x , up to and including the term in x^3 , the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

- (b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

- (c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

2.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

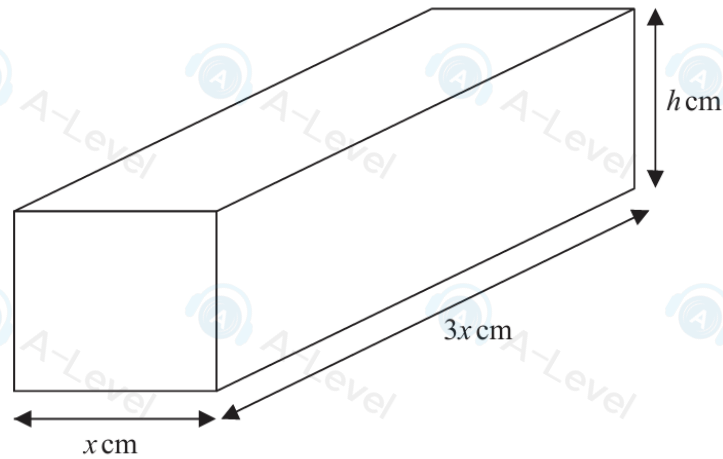


Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is 972 cm^3

(a) Show that the surface area of the brick, $S \text{ cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\frac{dS}{dx}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\frac{d^2S}{dx^2}$ and hence show that the value of x found in part (c) gives the minimum value of S .

(2)

(e) Hence find the minimum surface area of the brick.

(1)

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6.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \quad k \quad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the N th month.

(c) Calculate the value of N according to the model.

(3)

5.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \quad 0 \leq t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

10.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\cos \theta \left(3 \tan \theta + \frac{2}{\tan \theta} \right) \equiv \sin \theta + \frac{2}{\sin \theta} \quad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x \left(3 \tan x + \frac{2}{\tan x} \right) = 4 \sin x - 5$$

giving your answers to 3 significant figures.

(4)

4. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $f(x)$ is divided by $(x - 2)$ the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R .

(4)

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(ii) Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(5)

(c) Find the range of values of x for which $f(x)$ is decreasing.

(3)

4. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)