

8. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$

$$a_1 = p - 3$$

where p is a constant.

- (a) Find an expression for a_2 in terms of p , giving your answer in simplest form. (1)

Given that $\sum_{n=1}^3 a_n = p + 15$

- (b) find the possible values of a_2 (6)

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20} (20 + 6t - t^2) + 17 \quad 0 \leq t \leq 7$$

Given that H has a stationary value at $t = \alpha$

- (a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0 \quad (5)$$

- (b) Hence find the value of α , giving your answer to 3 decimal places. (1)

- (c) Use further calculus to prove that H is a maximum at this value of α . (2)

2. Find the first 3 terms in ascending powers of x of

$$\left(2 - \frac{x}{2}\right)^6$$

giving each term in its simplest form. (4)

(Total for Question 2 is 4 marks)

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1. The sequence $u_1, u_2, u_3, \dots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

- (a) find the value of u_2

(2)

- (b) find $\sum_{r=1}^4 (u_r + 2r)$

(3)

8. A metal post is repeatedly hit in order to drive it into the ground.

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Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

- (a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

- (b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

- (c) find the value of r , giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

- (d) Find, showing all steps in your working, the smallest possible value of N .

(4)

1.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = 1$$

$$u_{n+1} = k - \frac{8}{u_n} \quad n \geq 1$$

where k is a constant.

(a) Write down expressions for u_2 and u_3 in terms of k .

(2)

Given that $u_3 = 6$

(b) find the possible values of k .

(3)

(Total for Question 1 is 5 marks)

4. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$(3 + 2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{6x}\right)(3 + 2x)^6$$

(3)

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