5. $$\mathrm{f}(x) = 3x^3 + ax^2 - 10x + b$$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $\mathrm{f}(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $\mathrm{f}(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $\mathrm{f}(x)$.

(3)

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2 - x) = 4 + \log_2(x + 10)$$

(5)

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where a is a positive constant greater than 1

(1)

1. The continuous curve C has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below.

x	4.0	4.2	4.4	4.6	4.8	5.0
y	9.2	8.4556	3.8512	5.0342	7.8297	8.6

Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_4^5 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 3 decimal places.

(3)

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x+5) - \log_3(x+3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a+b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4"

(4)

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

DO NOT WRITE IN THIS AREA