6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

5. $$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51$$

(2)

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0$$

(2)

(c) Hence find the value of p and the value of q.

(3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x)$$

(2)

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7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$

(2)

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$

(2)

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$

(3)

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2}\log_a\left(y^r\right) = \sum_{r=1}^{2}\left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The binomial expansion, in ascending powers of x, of $(1 + kx)^n$ is

$$1 - 24x + 270x^2 + px^3 + \ldots$$

where k and p are constants and n is an integer greater than 1

(a) Use this information to set up and solve two simultaneous equations to find the value of k and the value of n.

(6)

(b) Hence find the value of p.

(2)

9: The circle C

- has a centre which lies on the x-axis
- touches the y-axis
- passes through the point (5, 6)

(a) On Diagram 1, sketch a graph of C.

(1)

(b) Find an equation for C.

(4)

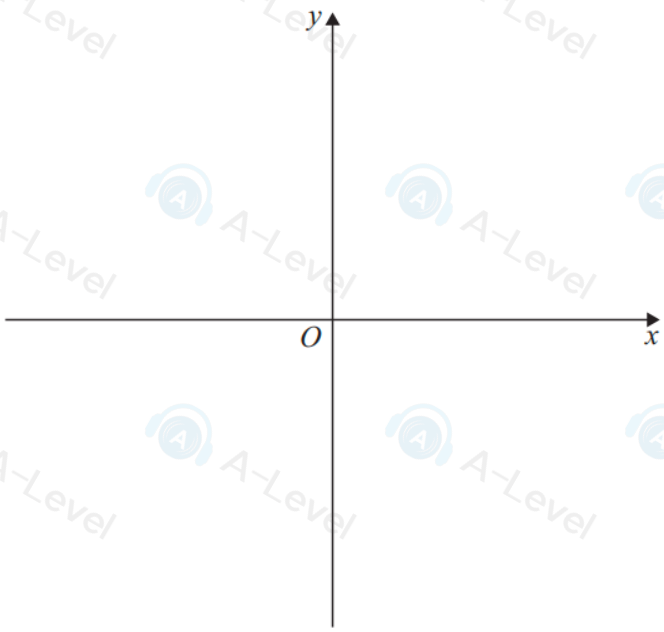

Diagram 1

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4. $f(x) = (x^2 - 2)(2x - 3) - 21$

(a) State the value of the remainder when $f(x)$ is divided by $(2x - 3)$

(1)

(b) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(c) Hence,

(i) factorise $f(x)$

(ii) show that the equation $f(x) = 0$ has only one real root.

(5)

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9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

2. One of the terms in the binomial expansion of $(3 + ax)^6$, where a is a constant, is $540x^4$

(a) Find the possible values of a.

(4)

(b) Hence find the term independent of x in the expansion of

$$\left(\frac{1}{81} + \frac{1}{x^6}\right)(3 + ax)^6$$

(3)

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

9. Given that

$$3\log_2(t + 4) - 2\log_2(t - 2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t + 4) - 2\log_2(t - 2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

3:

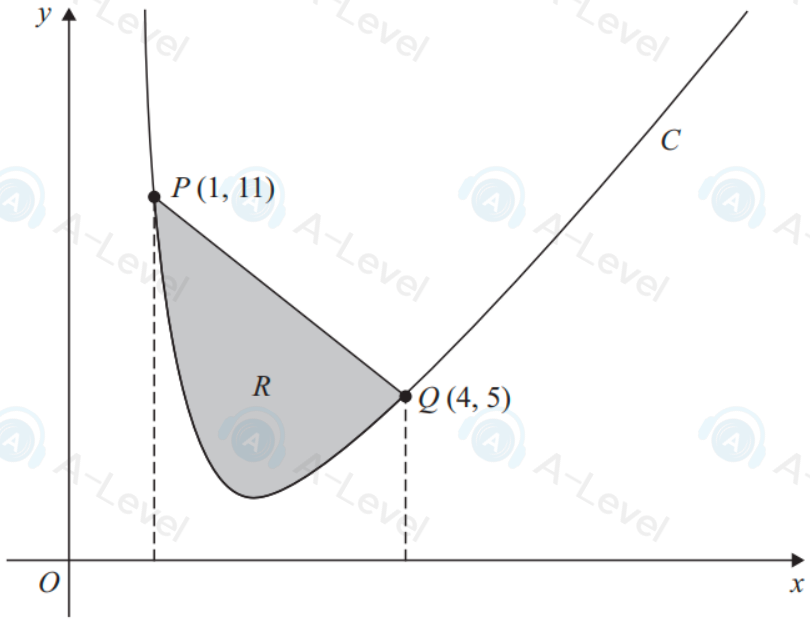

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with equation

$$y = 3x + \frac{16}{x^2} - 8 \qquad x > 0$$

The points $P(1, 11)$ and $Q(4, 5)$ lie on C and are shown in Figure 1.

The region R, shown shaded in Figure 1, is bounded by C and line segment PQ.

Use algebraic integration to find the area of R.

(5)

2. The table shows corresponding values of x and y for a continuous curve with equation $y = \mathrm{f}(x)$ between $x = -4$ and $x = 5$, where a is a constant.

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} \mathrm{f}(x)\,\mathrm{d}x$$

Given that the value of this approximation is 19.3

(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} \left(2\mathrm{f}(x) - 3\right)\mathrm{d}x$$

(2)