3.

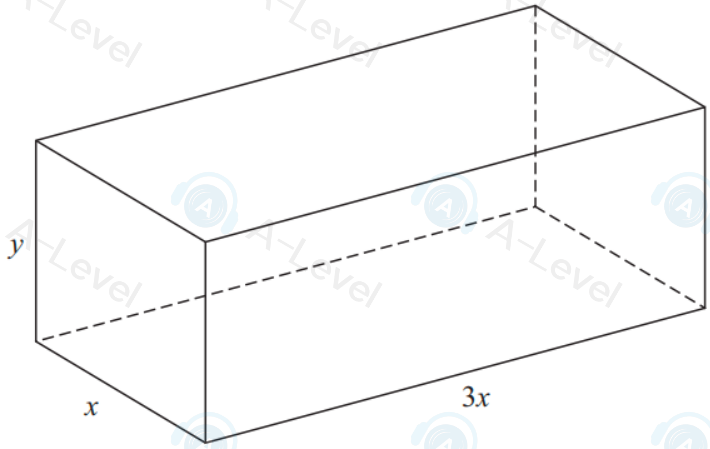

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

blank

10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

4.

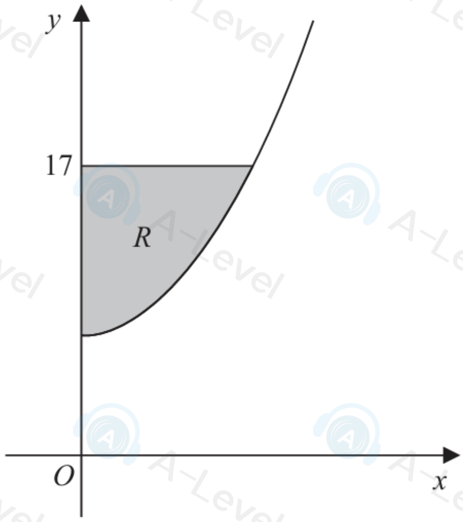

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = 2x^2 + 7 \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the line with equation $y = 17$

Find the exact area of R.

(6)

1.

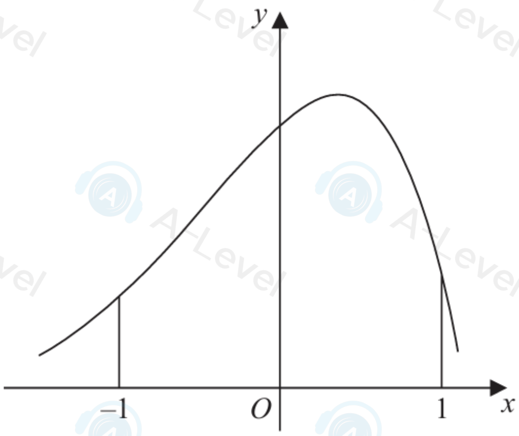

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$

The table below shows some corresponding values of x and y for this curve.

The values of y are given to 3 decimal places.

x	–1	–0.5	0	0.5	1
y	2.287	4.470	6.719	7.291	2.834

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to estimate

(i) $\displaystyle\int_{-1}^{1} \left(\mathrm{f}(x) - 2\right)\mathrm{d}x$

(ii) $\displaystyle\int_{1}^{3} \mathrm{f}(x-2)\,\mathrm{d}x$

(3)