8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a polynomial in x.

Given

$$\mathrm{f}'(x) = 2(3x - 2)(x + 5)$$

(a) deduce the range of values of x for which y is decreasing.

(2)

Given further that C cuts the x-axis at $\frac{7}{2}$

(b) state a factor of $\mathrm{f}(x)$, giving the answer in the form $(Ax + B)$ where A and B are integers.

(1)

(c) Hence find an expression for $\mathrm{f}(x)$ in a fully factorised form.

(6)

DO NOT WRITE IN THIS AREA

4:

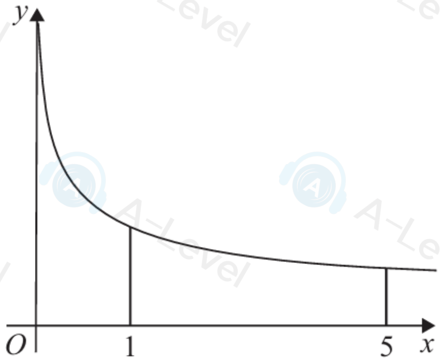

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \log_3 (x + 1) - \log_3 x$$

The point $P(a, 4)$ lies on the curve.

(a) Find the exact value of the constant a.

(*Solutions relying on calculator technology are not acceptable.*)

(4)

(b) Use the trapezium rule with 4 strips of equal width to estimate the value of

$$\int_1^5 \left(\log_3 (x+1) - \log_3 x\right) \mathrm{d}x$$

giving the answer in the form $\log_3 k$, where k is a constant to be found.

(4)

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

$$\int_1^5 \left(\log_3 (x+1) - \log_3 x\right) \mathrm{d}x$$

(1)

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

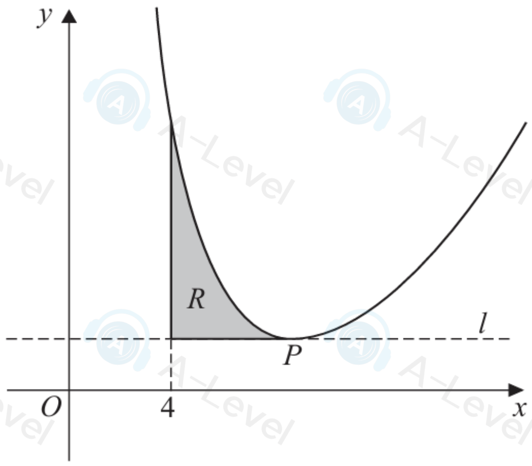

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

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9.

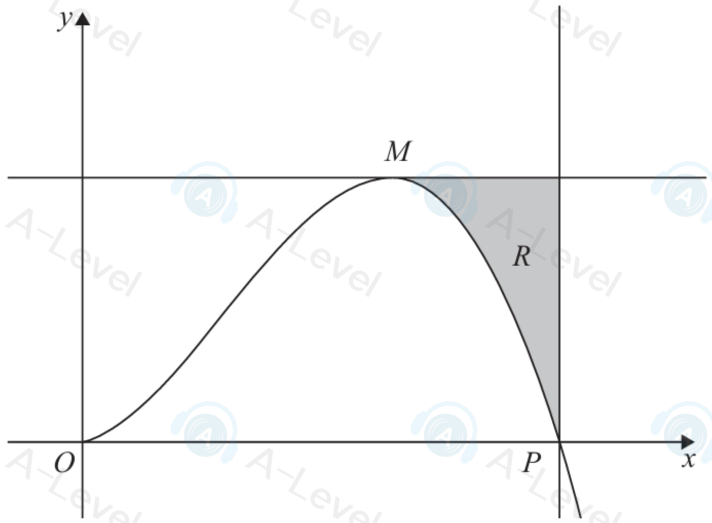

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

8. Solutions relying on calculator technology are not acceptable in this question.

(i)

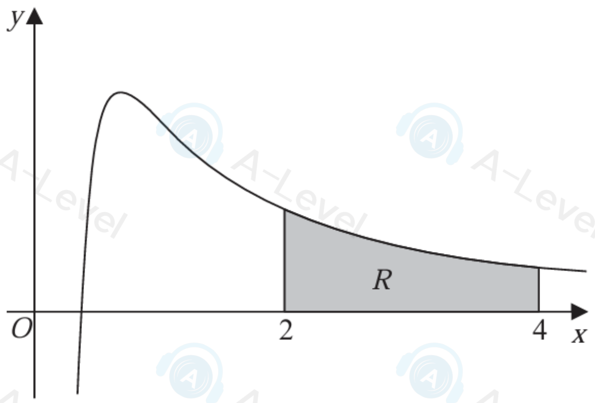

Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) \mathrm{d}x = 55$$

(4)

3.

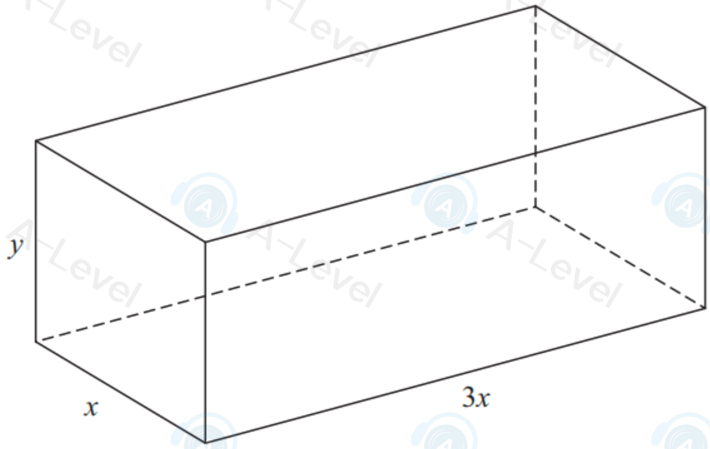

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)