

4.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

3. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

- (a) (i) a_1
(ii) a_2
(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50} \left\{ n + \cos^2\left(\frac{n\pi}{3}\right) \right\}$$

You must make your method clear.

(4)

1. The sequence $u_1, u_2, u_3, \dots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^4 (u_r + 2r)$

(3)

6. In a geometric sequence $u_1, u_2, u_3, \dots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

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7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A , find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B , find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan *A*, he will increase the distance he runs each Sunday by the same amount.

- (a) Calculate the distance he will run on Sunday in week 5 under training plan *A*. (3)

In training plan *B*, he will increase the distance he runs each Sunday by the same percentage.

- (b) Calculate the distance he will run on Sunday in week 5 under training plan *B*.
Give your answer in km to one decimal place. (3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
 - the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km
- (c) (i) find the maximum value of x , if he uses training plan *A*,
(ii) find the maximum value of x , if he uses training plan *B*. (5)

10.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

- (a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

- (b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

- (c) Find, according to the models, the value of T , giving your answer to one decimal place.

(3)