10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Given that

$$\log_a x + \log_a 3 = \log_a 27 - 1 \qquad \text{where } a \text{ is a positive constant}$$

find, in its simplest form, an expression for x in terms of a.

(4)

(ii) Solve the equation

$$(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$$

showing each step of your working.

(4)

(Total for Question 5 is 8 marks)

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2 (4 - x) = 3 + \log_2 \left(\frac{x + 11}{2} \right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \qquad \text{and} \qquad y = 6 \times 3^x$$

meet at the point P.

Find the exact coordinates of P, writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

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6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

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9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

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4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$

(2)

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$

(2)

(iii) $\sum_{n=1}^{50} (k + \log_a b^n)$

(3)

6.

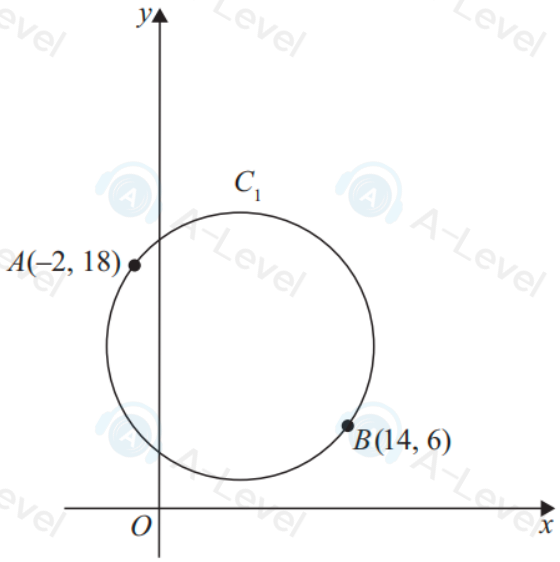

Figure 2

The points $A(-2, 18)$ and $B(14, 6)$ lie on a circle C_1 as shown in Figure 2.

Given that AB is a diameter of the circle C_1

(a) find an equation for C_1 making your method clear.

(5)

A circle C_2 has its centre at the origin.

Given that circles C_1 and C_2 touch,

(b) find possible equations for C_2

(4)

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9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x + 5) - \log_3(x + 3) = 2$$

(5)

(ii) Given that $a > 0, b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a + b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

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9. Given that

$$3\log_2(t + 4) - 2\log_2(t - 2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t + 4) - 2\log_2(t - 2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

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2: The line joining the points $(-2, 5)$ and $(4, 15)$ is the diameter of a circle C.

(a) Find an equation for C.

(5)

(b) Hence find the exact coordinates of the point on C that is nearest the x-axis.

(2)

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2. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$u_{n+1} = (-1)^n u_n + k$$

where k is a constant.

(a) Show that $u_5 = 7$

(3)

Given that $\sum_{r=1}^{4} u_r = 30$

(b) find the value of k.

(2)

(c) Hence find the value of $\sum_{r=1}^{150} u_r$

(2)

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