	5(a)	$\dfrac{dy}{dx} = \dfrac{(x^2+k) \times \dfrac{2x}{x^2+k} - 2x\ln(x^2+k)}{(x^2+k)^2}$	M1
		$\dfrac{dy}{dx} = \dfrac{2x - 2x\ln(x^2+k)}{(x^2+k)^2} = \dfrac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$	M1A1
			(3)
	(b)	$x = 0$	B1
		$"1" - \ln(x^2+k) = 0 \Rightarrow x^2 = e^{"1"} \pm k$	M1
		$x = \pm\sqrt{e-k}$	A1ft
			(3)
	(c)	Upper limit is e or $k < e$	B1ft
			(1)
			(7 marks)

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = ...$	M1
	$x = $ awrt $5.8°$, awrt $84.2°$	A1A1
		(3)
		(9 marks)

7 (a)

$$\sqrt{2}\sin(x+45°)=\cos(x-60°)$$

$$\sqrt{2}\left(\sin x\cos 45°+\cos x\sin 45°\right)=\cos x\cos 60°+\sin x\sin 60°$$ M1 A1

$$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$$

$$\cos x=\left(\sqrt{3}-2\right)\sin x$$ M1

$$\tan x\left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}\;*$$ A1*

(4)

(b) | States or uses $x+45°=2\theta$ o.e. | B1

Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$ M1

Correct order of operations to find one angle   dM1

$$\theta=75°,165°$$ A1

(4)

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x-\tan^2 x\equiv\dfrac{\cos^2 x}{\sin^2 x}-\dfrac{\sin^2 x}{\cos^2 x}\equiv\dfrac{\cos^4 x-\sin^4 x}{\sin^2 x\cos^2 x}$	M1
	$\equiv\dfrac{\left(\cos^2 x-\sin^2 x\right)\left(\cos^2 x+\sin^2 x\right)}{\sin^2 x\cos^2 x}\equiv\dfrac{\cos 2x}{\ldots}$ or $\dfrac{\ldots}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv\dfrac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\dfrac{\cos 2x}{\sin 2x\sin 2x}\equiv 4\cot 2x\operatorname{cosec}2x\;*$	A1*
		(4)
(b)	$4\cot 2\theta\operatorname{cosec}2\theta=2\tan^2\theta\Rightarrow\cot^2\theta-\tan^2\theta=2\tan^2\theta\Rightarrow\cot^2\theta-3\tan^2\theta=0$	M1
	$\cot^2\theta-3\tan^2\theta=0\Rightarrow\dfrac{1}{\tan^2\theta}-3\tan^2\theta=0\Rightarrow\tan^4\theta=\dfrac{1}{3}$	A1
	$\tan^4\theta="\dfrac{1}{3}"\Rightarrow\tan\theta=\pm\sqrt[4]{"\dfrac{1}{3}"}="\pm 0.7598.."\Rightarrow\theta=\ldots$	M1
	$\theta=\text{awrt }0.65,\;-0.65$	A1A1
		(5)
		Total 9

6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$	M1 A1
	$= 2(2x-3)^2 e^{4x-2}\{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	dM1 A1
		(4)
(b) (i)	$x = \dfrac{3}{2}, \dfrac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\dfrac{3}{4}\right) = \left(-\dfrac{3}{2}\right)^3 e^1 = -\dfrac{27}{8} e, \Rightarrow g\left(\dfrac{3}{4}\right) = -27e$	M1, A1
	$-27e \, _{,,} \, g(x) \, _{,,} \, 0 g \; g \; \text{?}$	A1
		(4)
		(8 marks)

3.	$\dfrac{dy}{dx} = \dfrac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2 - 2(4x+1)(x+3) = 0$	
	$\Rightarrow (x+3)(10-4x) = 0 \Rightarrow x = \ldots$	M1
	Critical value of $\dfrac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\dfrac{dy}{dx} > 0 \Rightarrow -3 < x < \dfrac{5}{2}$	A1
		(6 marks)

5.(a)	$\dfrac{dy}{dx} = -10\sin 2x - 24\cos 2x$	M1, A1
	$\left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{3}} = -10\sin\left(\dfrac{2\pi}{3}\right) - 24\cos\left(\dfrac{2\pi}{3}\right) = -5\sqrt{3} + 12$	A1
		(3)
(b)	$R = 13$	B1
	$\tan\alpha = \dfrac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	M1A1
		(3)
(c)	Sets $-26\sin(2x + 1.176) = 6$	M1
	$\sin(2x + "1.176") = -\dfrac{3}{13}$	A1 ft
	$x = \dfrac{\arcsin\left(-\dfrac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	dM1, A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
9(a)	$y = \sqrt{3 + 4e^{x^2}} = \left(3 + 4e^{x^2}\right)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\left(3 + 4e^{x^2}\right)^{-\frac{1}{2}} \times 8xe^{x^2}$	M1
	$= 4xe^{x^2}\left(3 + 4e^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\dfrac{\left(3 + 4e^{x^2}\right)^{\frac{1}{2}}}{x} = 4xe^{x^2}\left(3 + 4e^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\dfrac{\left(3 + 4e^{x^2}\right)}{x} = 4xe^{x^2}$	dM1
	$4x^2 e^{x^2} - 4e^{x^2} - 3 = 0 \; *$	A1*
		(3)
(c)	$f(x) = 4x^2 e^{x^2} - 4e^{x^2} - 3 \Rightarrow f(1) = -3 \quad \text{AND} \quad f(2) = 652\cdot...$	M1
	Change of sign and $f(x)$ is continuous hence root in $(1, 2)$	A1
		(2)
(d)	$4x^2 e^{x^2} - 4e^{x^2} - 3 = 0 \Rightarrow x^2 = \dfrac{4e^{x^2} + 3}{4e^{x^2}} = \dfrac{4 + 3e^{-x^2}}{4} \Rightarrow x = \dfrac{1}{2}\sqrt{4 + 3e^{-x^2}} \; *$	B1*
		(1)
(e)(i)	$x_1 = 1 \Rightarrow x_2 = \dfrac{1}{2}\sqrt{4 + 3e^{-1}}$	M1
	$x_3 = 1.0997$	A1
(ii)	$\alpha = 1.1051$	A1
		(3)
		Total 11

5 (i)		States $x = 2$	B1
		$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
		$x = \dfrac{5\pi}{6}$	A1
			(3)
(ii)		Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
		$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
		$\sin\theta = \dfrac{-5 \pm \sqrt{43}}{6} \left(= -1.926..., 0.2595...\right) \Rightarrow \theta = \arcsin(...)$	M1
		$\theta = 15.0°, \; 165°$	A1
			(4)
			(7 marks)

10.(a) $x = 3\cos 2y \Rightarrow \left(\dfrac{\mathrm{d}x}{\mathrm{d}y}\right) = -6\sin 2y$ M1 A1

(2)

(b) E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = -6\sqrt{1-kx^2}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\text{their } \dfrac{\mathrm{d}x}{\mathrm{d}y}}$ M1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{6\sqrt{1-\cos^2 2y}} = -\dfrac{1}{6\sqrt{1-\left(\dfrac{x}{3}\right)^2}} = -\dfrac{1}{2\sqrt{9-x^2}}$ dM1 A1

(3)

(c) Sets $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2\sqrt{9-x^2}} = -\dfrac{1}{4}$ M1 A1

$x^2 = 5 \Rightarrow a = \sqrt{5}$

$\sqrt{5} = 3\cos 2y \Rightarrow b = \dfrac{1}{2}\arccos\left(\dfrac{\sqrt{5}}{3}\right)$ dM1 A1

Alt (c) Sets $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{6\sin 2y} = -\dfrac{1}{4}$

$\Rightarrow \sin 2y = \dfrac{2}{3} \Rightarrow y = \dfrac{1}{2}\arcsin\left(\dfrac{2}{3}\right)$ M1A1

$x = 3\cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\dfrac{2}{3}\right)} = \sqrt{5}$ dM1 A1

(4)

(9 marks)

Question Number	Scheme	Marks
6(i)	$\left(\dfrac{dy}{dx}=\right)\dfrac{4x}{2x^2+5}$	M1A1
		(2)
(ii)	$\displaystyle\int\dfrac{21x}{3x^2+k}\,dx=\dfrac{7}{2}\ln(3x^2+k)\ (+c)$	M1A1
	$\left[\dfrac{7}{2}\ln(3x^2+k)\right]_1^k<7\ln 8\Rightarrow\dfrac{7}{2}\ln\left(\dfrac{3k^2+k}{3+k}\right)<7\ln 8\Rightarrow\dfrac{3k^2+k}{3+k}<64$	M1
	$\dfrac{3k^2+k}{3+k}(<)64\Rightarrow 3k^2-63k-192\ (<0)\Rightarrow k=...$	dM1
	$k=23$	A1
		(5)
		(7 marks)