

$$f(x) = 8\sin x \cos x + 4\cos^2 x - 3$$

4(a)

States or uses  $\sin 2x = 2\sin x \cos x$  or  $\cos 2x = \pm 2\cos^2 x \pm 1$

M1

Uses  $\sin 2x = 2\sin x \cos x$  and  $\cos 2x = \pm 2\cos^2 x \pm 1$  in  $f(x)$

dM1

$$(f(x) =) 8\sin x \cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$$

A1

(3)

(b)

$$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$$

B1ft

$$\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots \text{ ("awrt 0.464")}$$

M1

$$(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$$

A1

(3)

(c)

(i) Maximum value = " $2\sqrt{5} - 1$ "

B1 ft

(ii) Solves  $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$

M1

$$(x =) \text{awrt 3.69 (or } (x =) \text{awrt 3.70)}$$

A1

(3)

(9 marks)

Question Number	Scheme	Marks
4(a)	$R = \sqrt{12} \text{ or } 2\sqrt{3}$	B1
	$\alpha = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \dots$	M1
	$(f(x) =) \sqrt{12} \sin\left(2x - \frac{\pi}{3}\right)$	A1
		(3)
(b)(i)	Minimum value = $\frac{18}{\sqrt{12} + 4\sqrt{3}} (= \sqrt{3})$	B1
(ii)	$\sin\left(6x - \frac{\pi}{3}\right) = 1 \Rightarrow x = \frac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

<p>5 (i)</p>	<p>States <math>x = 2</math></p> $\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ $x = \frac{5\pi}{6}$	<p>B1 M1 A1 (3)</p>
<p>(ii)</p>	<p>Attempts to use <math>\cos 2\theta = 1 - 2\sin^2\theta</math></p> $6\sin^2\theta + 10\sin\theta - 3 = 0$ $\sin\theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$ $\theta = 15.0^\circ, 165^\circ$	<p>M1 A1 M1 A1 (4) (7 marks)</p>
<p>7 (a)</p>	$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$ $\sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$ $\sin x + \cos x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$ $\cos x = (\sqrt{3} - 2) \sin x$ $\tan x \left( = \frac{1}{\sqrt{3} - 2} = \frac{\sqrt{3} + 2}{-1} \right) = -2 - \sqrt{3} \quad *$	<p>M1 A1 M1 A1* (4)</p>
<p>(b)</p>	<p>States or uses <math>x + 45^\circ = 2\theta</math> o.e.</p> <p>Proceeds from e.g. <math>\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta - 45^\circ = 105^\circ, 285^\circ</math></p> <p>Correct order of operations to find one angle</p> $\theta = 75^\circ, 165^\circ$	<p>B1 M1 dM1 A1 (4)</p>
<p>7 (a)</p>	$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$ $\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$ $\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$ $\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$	<p>M1 dM1 A1 (3)</p>
<p>(b)</p>	$\sec x \sin 4x = 5 \sin^3 x \cot x$ $\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$ $\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$ $\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$ $\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0 \quad *$	<p>B1 M1 A1* (3)</p>
<p>(c)</p>	<p>Uses <math>\sec^2 x = 1 + \tan^2 x</math></p> $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ $\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$ $\Rightarrow x = \text{awrt } 0.506, 2.08$	<p>M1 A1 dM1 A1 (4) (10 marks)</p>

<b>8(a)</b>	Starting with the LHS: $2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) = \frac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \frac{2 - 2(1 - 2\sin^2 \theta)}{4\sin^2 \theta \cos^2 \theta}$	M1dM1
	$= \sec^2 \theta = 1 + \tan^2 \theta \equiv \text{RHS} \quad *$	A1*
		<b>(4)</b>
<b>(b)</b>	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = \dots$	M1
	$\cos x = \frac{1}{4} \quad (\text{ignore } -1)$	A1
	$\cos x = \frac{1}{4} \Rightarrow x = \dots$	dM1
	$x = 75.5^\circ, 284.5^\circ$	A1
		<b>(4)</b>
	<b>(8 marks)</b>	

<b>8(a)</b>	$\tan 3x \equiv \tan(2x + x) \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x + x)}{\cos(2x + x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1
	$\frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \equiv \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$ $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \quad *$	A1*
		<b>(3)</b>

<b>(b)</b>	$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2 \sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2 \tan^2 3\theta - \tan 3\theta - 6 = 0 \quad \text{or e.g.} \Rightarrow 2 \tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2 \tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots \quad \text{or} \quad \tan 3\theta = 2 \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		<b>(5)</b>
	<b>Total 8</b>	

Question Number	Scheme	Marks
<b>9(a)</b>	$4 \sin \theta \cos \theta = 2 \sin 2\theta$	B1
	e.g. $\Rightarrow 6 \sin^2 \theta \cot 2\theta + 2 \sin 2\theta = (3 - 3 \cos 2\theta) \frac{\cos 2\theta}{\sin 2\theta} + 2 \sin 2\theta$	M1A1 <b>(3)</b>
<b>(b)</b>	$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$	
	e.g. $\Rightarrow 3 \cot 2\theta \sin 2\theta - 14 \sin 2\theta = (3 - 3 \cos 2\theta) \cos 2\theta + 2 \sin^2 2\theta$	M1
	$\Rightarrow -14 \sin 2\theta = -3(1 - \sin^2 2\theta) + 2 \sin^2 2\theta$	M1
	$5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0$ *	A1*
		<b>(3)</b>
<b>(c)</b>	$(\sin 2x =) \frac{1}{5} \Rightarrow x = \dots$	M1
	$x = \text{awrt } 5.8^\circ, \text{ awrt } 84.2^\circ$	A1A1
		<b>(3)</b>
		<b>(9 marks)</b>

<b>2(a)</b>	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		<b>(3)</b>
<b>(b)(i)</b>	$\text{Min} = \frac{5}{90 - 3 \times "25" \times (-1)}$	M1
	$= \frac{1}{33}$	A1
<b>(b)(ii)</b>	$(2x + "1.287") = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - "1.287"}{2} = 0.927$	A1
		<b>(4)</b>
		<b>Total 7</b>

Question Number	Scheme	Marks
<b>8 (i)</b>	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ Uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ $3 \operatorname{cosec} \theta = 8 \cos \theta \Rightarrow \sin 2\theta = \frac{3}{4}$ $\Rightarrow \theta = \frac{1}{2} \arcsin\left(\frac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	B1 M1 A1 M1 A1 <b>(5)</b>
<b>(ii)</b>	$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\frac{3}{8} \Rightarrow \tan(2x - 70^\circ) = -\frac{3}{8}$ $\arctan\left(-\frac{3}{8}\right) + 70^\circ$ Correct order of operations $x = \frac{\arctan\left(-\frac{3}{8}\right) + 70^\circ}{2}$ awrt $24.7^\circ$ , awrt $114.7^\circ$	M1 A1 dM1 A1 <b>(4)</b>
		<b>(9 marks)</b>

Question Number	Scheme	Marks
<b>9(a)</b>	$\frac{3 \sin \theta \cos \theta}{\cos \theta + \sin \theta} = (2 + \sec 2\theta)(\cos \theta - \sin \theta)$ $\Rightarrow \frac{3}{2} \sin 2\theta = (2 + \sec 2\theta)(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)$ <b>or</b> $\Rightarrow 3 \sin \theta \cos \theta = (2 + \sec 2\theta) \cos 2\theta$	M1
	$\Rightarrow \frac{3}{2} \sin 2\theta = (2 + \sec 2\theta) \cos 2\theta$	<b>dM1</b>
	$\Rightarrow \frac{3}{2} \sin 2\theta = (2 + \sec 2\theta) \cos 2\theta$ $\Rightarrow \frac{3}{2} \sin 2\theta = 2 \cos 2\theta + 1 \Rightarrow 3 \sin 2\theta - 4 \cos 2\theta = 2^*$	A1*
		<b>(3)</b>

<b>(b)</b>	Way 1 using $R \sin(2x - \alpha)$ or $R \cos(2x + \alpha)$	
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots(5)$ <b>or</b> $(\alpha =) \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right) = \dots$	M1
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots(5)$ <b>and</b> $(\alpha =) \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right) = \dots$	<b>dM1</b>
	$3 \sin 2x - 4 \cos 2x = 2 \Rightarrow 5 \sin(2x - 0.927) = 2$ <b>or</b> $3 \sin 2x - 4 \cos 2x = 2 \Rightarrow 5 \cos(2x + 0.644) = -2$	A1
	$5 \sin(2x - 0.927) = 2 \Rightarrow x = \frac{\sin^{-1} \frac{2}{5} + 0.927}{2}$ <b>or</b> $5 \cos(2x + 0.644) = -2 \Rightarrow x = \frac{\cos^{-1} \frac{-2}{5} - 0.644}{2}$	<b>ddM1</b>
	$x = \text{awrt } 3.81$	A1
		<b>(5)</b>
		<b>(8 marks)</b>