Question Number	Scheme	Marks
4(a)	$R = \sqrt{12}$ or $2\sqrt{3}$	B1
	$\alpha = \tan^{-1}\left(\dfrac{3}{\sqrt{3}}\right) = \ldots$	M1
	$(f(x) =) \sqrt{12}\sin\left(2x - \dfrac{\pi}{3}\right)$	A1
		(3)
(b)(i)	Minimum value $= \dfrac{18}{\sqrt{12} + 4\sqrt{3}}\left(= \sqrt{3}\right)$	B1
(ii)	$\sin\left(6x - \dfrac{\pi}{3}\right) = 1 \Rightarrow x = \dfrac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = $ awrt 0.424, awrt 1.15	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\dfrac{3}{8} \Rightarrow \tan\left(2x - 70°\right) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right) + 70°}{2}$	dM1
	awrt 24.7° , awrt 114.7°	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
9(a)	$$\frac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta}=(2+\sec 2\theta)(\cos\theta-\sin\theta)$$ $$\Rightarrow \frac{3}{2}\sin 2\theta=(2+\sec 2\theta)(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)$$ **or** $$\Rightarrow 3\sin\theta\cos\theta=(2+\sec 2\theta)\cos 2\theta$$	M1
	$$\Rightarrow \frac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$$	**d**M1
	$$\Rightarrow \frac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$$ $$\Rightarrow \frac{3}{2}\sin 2\theta=2\cos 2\theta+1\Rightarrow 3\sin 2\theta-4\cos 2\theta=2 \text{ *}$$	A1*
		(3)

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$$\Rightarrow R=\sqrt{3^2+4^2}=...(5)\ \textbf{or}\ (\alpha=)\ \tan^{-1}\left(\tfrac{4}{3}\text{ or }\tfrac{3}{4}\right)=...$$	M1
	$$\Rightarrow R=\sqrt{3^2+4^2}=...(5)\ \textbf{and}\ (\alpha=)\ \tan^{-1}\left(\tfrac{4}{3}\text{ or }\tfrac{3}{4}\right)=...$$	**d**M1
	$$3\sin 2x-4\cos 2x=2\Rightarrow 5\sin(2x-0.927)=2$$ **or** $$3\sin 2x-4\cos 2x=2\Rightarrow 5\cos(2x+0.644)=-2$$	A1
	$$5\sin(2x-0.927)=2\Rightarrow x=\frac{\sin^{-1}\frac{2}{5}+0.927}{2}$$ **or** $$5\cos(2x+0.644)=-2\Rightarrow x=\frac{\cos^{-1}\frac{-2}{5}-0.644}{2}$$	**dd**M1
	$$x=\text{awrt }3.81$$	A1
		(5)
		(8 marks)

8(a)	$$\tan 3x\equiv\tan(2x+x)\equiv\frac{\tan 2x+\tan x}{1-\tan 2x\tan x}$$ or e.g. $$\tan 3x\equiv\frac{\sin 3x}{\cos 3x}\equiv\frac{\sin(2x+x)}{\cos(2x+x)}\equiv\frac{\sin 2x\cos x+\cos 2x\sin x}{\cos 2x\cos x-\sin 2x\sin x}\equiv\frac{\tan 2x+\tan x}{1-\tan 2x\tan x}$$	M1
	$$\frac{\tan 2x+\tan x}{1-\tan 2x\tan x}\equiv\frac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$$	dM1
	$$\frac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}\equiv\frac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x-2\tan^2 x}$$ $$\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}\text{ *}$$	A1*
		(3)

(b)	$\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = 2\sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2\left(1+\tan^2 3\theta\right) - 8$	M1
	$\Rightarrow 2\tan^2 3\theta - \tan 3\theta - 6 = 0$ or e.g. $\Rightarrow 2\tan^2 3\theta - \tan 3\theta = 6$	A1
	$\left(2\tan 3\theta + 3\right)\left(\tan 3\theta - 2\right) = 0 \Rightarrow \tan 3\theta = -\dfrac{3}{2},\ 2$ $\tan 3\theta = -\dfrac{3}{2} \Rightarrow 3\theta = ... \Rightarrow \theta = ...$ or $\tan 3\theta = 2 \Rightarrow 3\theta = ... \Rightarrow \theta = ...$	dM1
	$\theta = 0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

5 (i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
	$x = \dfrac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \dfrac{-5 \pm \sqrt{43}}{6}\ (=-1.926...,\ 0.2595...) \Rightarrow \theta = \arcsin(...)$	M1
	$\theta = 15.0°,\ 165°$	A1
		(4)
		(7 marks)

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = \left(3-3\cos 2\theta\right)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = \left(3-3\cos 2\theta\right)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3\left(1-\sin^2 2\theta\right) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$\left(\sin 2x =\right)\dfrac{1}{5} \Rightarrow x = ...$	M1
	$x = $ awrt $5.8°$, awrt $84.2°$	A1A1
		(3)
		(9 marks)

1	$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\left(\sec^2\theta - 1\right) + 7\sec\theta - 3 = 0$		M1
	$3\sec^2\theta + 7\sec\theta - 6 = 0$		A1
	$\left(3\sec\theta - 2\right)\left(\sec\theta + 3\right) = 0 \Rightarrow \sec\theta = \ldots \Rightarrow \cos\theta = \ldots$		dM1
	$\theta = 109.5°, 250.5°$		A1, A1
			(5)

2(a)	$R = 25$		B1
	$\tan\alpha = \dfrac{24}{7} \Rightarrow \alpha = \ldots$		M1
	$\alpha = 1.287$		A1
			(3)
(b)(i)	$\text{Min} = \dfrac{5}{90 - 3 \times "25" \times (-1)}$		M1
	$= \dfrac{1}{33}$		A1
(b)(ii)	$\left(2x + "1.287"\right) = \pi, \ldots \Rightarrow x = \ldots$		M1
	$\Rightarrow x = \dfrac{\pi - "1.287"}{2} = 0.927$		A1
			(4)
			Total 7

7 (a)	$\sin 4\theta \equiv 2\sin 2\theta \cos 2\theta$		M1
	$\equiv 2\left(2\sin\theta\cos\theta\right)\left(1 - 2\sin^2\theta\right)$		dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$		
	$\equiv \sin\theta\cos\theta\left(4 - 8\sin^2\theta\right)$		A1
			(3)
(b)	$\sec x \sin 4x = 5\sin^3 x \cot x$		
	$\dfrac{1}{\cos x} \times \cos x \sin x\left(4 - 8\sin^2 x\right) = 5\sin^3 x \cot x$		B1
	$\div \sin x \Rightarrow 4 - 8\sin^2 x = 5\sin^2 x \cot x$		
	$\div \cos^2 x \Rightarrow 4\sec^2 x - 8\tan^2 x = 5\tan^2 x \cot x$		M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0 \qquad *$		A1*
			(3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$		M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$		A1
	$\Rightarrow \tan x = \dfrac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \ldots$		dM1
	$\Rightarrow x = \text{awrt } 0.506, 2.08$		A1
			(4)
			(10 marks)

7 (a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$		
	$\sqrt{2}\left(\sin x\cos 45°+\cos x\sin 45°\right)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1	
	$\sin x+\cos x=\dfrac{1}{2}\cos x+\dfrac{\sqrt{3}}{2}\sin x$		
	$\cos x=\left(\sqrt{3}-2\right)\sin x$	M1	
	$\tan x\left(=\dfrac{1}{\sqrt{3}-2}=\dfrac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}\quad *$	A1*	
			(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1	
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1	
	Correct order of operations to find one angle	dM1	
	$\theta=75°,165°$	A1	
			(4)

	$f(x)=8\sin x\cos x+4\cos^2 x-3$		
4(a)	States or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$	M1	
	Uses $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ in $f(x)$	dM1	
	$\left(f(x)=\right)8\sin x\cos x+4\cos^2 x-3=4\sin 2x+2\cos 2x-1$	A1	
			(3)
(b)	$R^2=a^2+b^2\Rightarrow R=\sqrt{20}$ or $2\sqrt{5}$	B1ft	
	$\tan\alpha=\dfrac{b}{a}\Rightarrow\alpha=...\ \left(="\text{awrt }0.464"\right)$	M1	
	$\left(f(x)=\right)2\sqrt{5}\sin\left(2x+0.464\right)-1$	A1	
			(3)
(c)	(i) Maximum value $="2\sqrt{5}-1"$	B1 ft	
	(ii) Solves $2x+\alpha=\dfrac{5\pi}{2}\Rightarrow x=...$	M1	
	$\left(x=\right)$ awrt 3.69 (or $\left(x=\right)$ awrt 3.70)	A1	
			(3)
			(9 marks)