Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv \dfrac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \dfrac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \dfrac{\cos 2x}{...}$ or $\dfrac{...}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \dfrac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\dfrac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x \operatorname{cosec} 2x$ *	A1*
		(4)
(b)	$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - \tan^2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - 3\tan^2\theta = 0$	M1
	$\cot^2\theta - 3\tan^2\theta = 0 \Rightarrow \dfrac{1}{\tan^2\theta} - 3\tan^2\theta = 0 \Rightarrow \tan^4\theta = \dfrac{1}{3}$	A1
	$\tan^4\theta = "\frac{1}{3}" \Rightarrow \tan\theta = \pm\sqrt[4]{"\frac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = ...$	M1
	$\theta =$ awrt $0.65, \ -0.65$	A1A1
		(5)
		Total 9

8(a)	$\tan 3x \equiv \tan(2x + x) \equiv \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \dfrac{\dfrac{2\tan x}{1 - \tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1 - \tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1 - \tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1 - \tan^2 x}\tan x} \equiv \dfrac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2\tan^2 x}$ $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ *	A1*
		(3)

(b)	$\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2\left(1+\tan^2 3\theta\right)-8$	M1
	$\Rightarrow 2\tan^2 3\theta - \tan 3\theta - 6 = 0$ or e.g. $\Rightarrow 2\tan^2 3\theta - \tan 3\theta = 6$	A1
	$\left(2\tan 3\theta+3\right)\left(\tan 3\theta-2\right)=0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = ... \Rightarrow \theta = ...$ or $\tan 3\theta = 2 \Rightarrow 3\theta = ... \Rightarrow \theta = ...$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
		Total 8

Question Number	Scheme	Marks
9(a)	$\frac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta}=(2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)(\cos\theta-\sin\theta)\left(\cos\theta+\sin\theta\right)$ **or** $\Rightarrow 3\sin\theta\cos\theta = (2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$	**d**M1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \frac{3}{2}\sin 2\theta = 2\cos 2\theta + 1 \Rightarrow 3\sin 2\theta - 4\cos 2\theta = 2$ *	A1*
		(3)

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **or** $(\alpha =)\ \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	M1
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **and** $(\alpha =)\ \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	**d**M1
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 5\sin(2x-0.927)=2$ **or** $3\sin 2x - 4\cos 2x = 2 \Rightarrow 5\cos(2x+0.644)=-2$	A1
	$5\sin(2x-0.927)=2 \Rightarrow x = \frac{\sin^{-1}\frac{2}{5}+0.927}{2}$ **or** $5\cos(2x+0.644)=-2 \Rightarrow x = \frac{\cos^{-1}\frac{-2}{5}-0.644}{2}$	**dd**M1
	$x =$ awrt 3.81	A1
		(5)
		(8 marks)

5 (i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
	$x = \frac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926..., 0.2595...) \Rightarrow \theta = \arcsin(...)$	M1
	$\theta = 15.0°, 165°$	A1
		(4)
		(7 marks)

7 (a)	$\sin 4\theta \equiv 2\sin 2\theta \cos 2\theta$	M1
	$\equiv 2(2\sin\theta\cos\theta)(1 - 2\sin^2\theta)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta(4 - 8\sin^2\theta)$	A1
		(3)
(b)	$\sec x \sin 4x = 5\sin^3 x \cot x$	
	$\frac{1}{\cos x} \times \cos x \sin x(4 - 8\sin^2 x) = 5\sin^3 x \cot x$	B1
	$\div \sin x \Rightarrow\ 4 - 8\sin^2 x = 5\sin^2 x \cot x$	
	$\div \cos^2 x \Rightarrow\ 4\sec^2 x - 8\tan^2 x = 5\tan^2 x \cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$ *	A1*
		(3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = ...$	dM1
	$\Rightarrow x =$ awrt 0.506, 2.08	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = ...$	M1
	$x =$ awrt $5.8°$, awrt $84.2°$	A1A1
		(3)
		(9 marks)

1	$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\left(\sec^2\theta - 1\right) + 7\sec\theta - 3 = 0$	M1
	$3\sec^2\theta + 7\sec\theta - 6 = 0$	A1
	$(3\sec\theta - 2)(\sec\theta + 3) = 0 \Rightarrow \sec\theta = ... \Rightarrow \cos\theta = ...$	dM1
	$\theta = 109.5°, 250.5°$	A1, A1
		(5)

Question Number	Scheme	Marks
4(a)	$R=\sqrt{12}$ or $2\sqrt{3}$	B1
	$\alpha=\tan^{-1}\left(\frac{3}{\sqrt{3}}\right)=\ldots$	M1
	$(\mathrm{f}(x)=)\sqrt{12}\sin\left(2x-\frac{\pi}{3}\right)$	A1
		(3)
(b)(i)	Minimum value $=\frac{18}{\sqrt{12}+4\sqrt{3}}\left(=\sqrt{3}\right)$	B1
(ii)	$\sin\left(6x-\frac{\pi}{3}\right)=1\Rightarrow x=\frac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

	$\mathrm{f}(x)=8\sin x\cos x+4\cos^2 x-3$	
4(a)	States or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$	M1
	Uses $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ in $\mathrm{f}(x)$	dM1
	$(\mathrm{f}(x)=)\ 8\sin x\cos x+4\cos^2 x-3=4\sin 2x+2\cos 2x-1$	A1
		(3)
(b)	$R^2=a^2+b^2\Rightarrow R=\sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha=\frac{b}{a}\Rightarrow\alpha=\ldots$ $(=\text{"awrt 0.464"})$	M1
	$(\mathrm{f}(x)=)\ 2\sqrt{5}\sin(2x+0.464)-1$	A1
		(3)
(c)	(i) Maximum value = "$2\sqrt{5}-1$"	B1 ft
	(ii) Solves $2x+\alpha=\frac{5\pi}{2}\Rightarrow x=\ldots$	M1
	$(x=)$ awrt 3.69 (or $(x=)$ awrt 3.70)	A1
		(3)
		(9 marks)

7 (a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$		
	$\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1	
	$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$		
	$\cos x=(\sqrt{3}-2)\sin x$	M1	
	$\tan x\left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*	
			(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1	
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1	
	Correct order of operations to find one angle	dM1	
	$\theta=75°,165°$	A1	
			(4)

5(a)	$\sin 3x=\sin(2x+x)=\sin 2x\cos x+\cos 2x\sin x$	M1
	$=(2\sin x\cos x)\cos x+(1-2\sin^2 x)\sin x$	dM1
	$=2\sin x\cos^2 x+\sin x-2\sin^3 x$	A1
	$=2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x=3\sin x-4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta=5\sin 2\theta\Rightarrow 2(3\sin\theta-4\sin^3\theta)=10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta=1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta-8\sin^3\theta=10\sin\theta\cos\theta\Rightarrow 6-8(1-\cos^2\theta)=10\cos\theta$ $\Rightarrow 4\cos^2\theta-5\cos\theta-1=0$ $\Rightarrow \cos\theta=\frac{5-\sqrt{41}}{8}=(-0.175...)$	dM1
	Any two of the following four answers $\sin\theta=0\Rightarrow\theta=180°,360°$ $\cos\theta=\frac{5-\sqrt{41}}{8}\Rightarrow\theta=$ awrt 100° or awrt 260°	A1
	All of 180°, 360°, awrt 100.1°, awrt 259.9°	A1
		(4)
		Total 8