

Question Number	Scheme	Marks
9(a)	$4 \sin \theta \cos \theta = 2 \sin 2\theta$	B1
	e.g. $\Rightarrow 6 \sin^2 \theta \cot 2\theta + 2 \sin 2\theta = (3 - 3 \cos 2\theta) \frac{\cos 2\theta}{\sin 2\theta} + 2 \sin 2\theta$	M1A1 (3)
(b)	$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$	
	e.g. $\Rightarrow 3 \cot 2\theta \sin 2\theta - 14 \sin 2\theta = (3 - 3 \cos 2\theta) \cos 2\theta + 2 \sin^2 2\theta$	M1
	$\Rightarrow -14 \sin 2\theta = -3(1 - \sin^2 2\theta) + 2 \sin^2 2\theta$	M1
	$5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =) \frac{1}{5} \Rightarrow x = \dots$	M1
	$x = \text{awrt } 5.8^\circ, \text{ awrt } 84.2^\circ$	A1A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\dots} \text{ or } \frac{\dots}{\left(\frac{1}{2} \sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2} \sin 2x\right)^2}$	A1
	$\equiv 4 \frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4 \cot 2x \operatorname{cosec} 2x$ *	A1*
		(4)
(b)	$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - \tan^2 \theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - 3 \tan^2 \theta = 0$	M1
	$\cot^2 \theta - 3 \tan^2 \theta = 0 \Rightarrow \frac{1}{\tan^2 \theta} - 3 \tan^2 \theta = 0 \Rightarrow \tan^4 \theta = \frac{1}{3}$	A1
	$\tan^4 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \sqrt[4]{\frac{1}{3}} = \pm 0.7598.. \Rightarrow \theta = \dots$	M1
	$\theta = \text{awrt } 0.65, -0.65$	A1A1
		(5)
		Total 9

2(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90 - 3 \times 25 \times (-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x + 1.287) = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - 1.287}{2} = 0.927$	A1
		(4)
		Total 7

Question Number	Scheme	Marks
4(a)	$R = \sqrt{12} \text{ or } 2\sqrt{3}$	B1
	$\alpha = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \dots$	M1
	$(f(x) =) \sqrt{12} \sin\left(2x - \frac{\pi}{3}\right)$	A1
		(3)
(b)(i)	Minimum value = $\frac{18}{\sqrt{12} + 4\sqrt{3}} (= \sqrt{3})$	B1
(ii)	$\sin\left(6x - \frac{\pi}{3}\right) = 1 \Rightarrow x = \frac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

7 (a)	$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$ $\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$ $\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$ $\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$	M1 dM1 A1 (3)
(b)	$\sec x \sin 4x = 5 \sin^3 x \cot x$ $\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$ $\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$ $\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$ $\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0 \quad *$	B1 M1 A1* (3)
(c)	<p>Uses $\sec^2 x = 1 + \tan^2 x$</p> $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ $\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$ $\Rightarrow x = \text{awrt } 0.506, 2.08$	M1 A1 dM1 A1 (4)



(10 marks)

5(a)	$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1
	$= (2 \sin x \cos x) \cos x + (1 - 2 \sin^2 x) \sin x$	dM1
	$= 2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$	A1
	$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x$	A1
	(4)	
(b)	$2 \sin 3\theta = 5 \sin 2\theta \Rightarrow 2(3 \sin \theta - 4 \sin^3 \theta) = 10 \sin \theta \cos \theta$	M1
	<p>Divides or takes out $\sin \theta$ as a factor and uses $\sin^2 \theta = 1 - \cos^2 \theta$ to set up and solve a 3TQ in $\cos \theta$</p> <p>E.g. $\Rightarrow 6 \sin \theta - 8 \sin^3 \theta = 10 \sin \theta \cos \theta \Rightarrow 6 - 8(1 - \cos^2 \theta) = 10 \cos \theta$</p> $\Rightarrow 4 \cos^2 \theta - 5 \cos \theta - 1 = 0$ $\Rightarrow \cos \theta = \frac{5 - \sqrt{41}}{8} = (-0.175\dots)$	dM1
	<p>Any two of the following four answers</p> $\sin \theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ $\cos \theta = \frac{5 - \sqrt{41}}{8} \Rightarrow \theta = \text{awrt } 100^\circ \text{ or awrt } 260^\circ$	A1
	<p>All of $180^\circ, 360^\circ, \text{awrt } 100.1^\circ, \text{awrt } 259.9^\circ$</p>	A1
	(4)	

Total 8

8(a)	Starting with the LHS: $2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) = \frac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \frac{2 - 2(1 - 2\sin^2 \theta)}{4\sin^2 \theta \cos^2 \theta}$	M1dM1
	$= \sec^2 \theta = 1 + \tan^2 \theta \equiv \text{RHS} \quad *$	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = \dots$	M1
	$\cos x = \frac{1}{4} \quad (\text{ignore } -1)$	A1
	$\cos x = \frac{1}{4} \Rightarrow x = \dots$	dM1
	$x = 75.5^\circ, 284.5^\circ$	A1
		(4)
		(8 marks)

7 (a)	$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$	M1 A1 M1 A1* (4)
	$\sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$	
	$\sin x + \cos x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$	
	$\cos x = (\sqrt{3} - 2) \sin x$	
	$\tan x \left(= \frac{1}{\sqrt{3} - 2} = \frac{\sqrt{3} + 2}{-1} \right) = -2 - \sqrt{3} \quad *$	
(b)	States or uses $x + 45^\circ = 2\theta$ o.e. Proceeds from e.g. $\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta - 45^\circ = 105^\circ, 285^\circ$ Correct order of operations to find one angle $\theta = 75^\circ, 165^\circ$	B1 M1 dM1 A1 (4)

5 (i)	States $x = 2$	B1
	$\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ $x = \frac{5\pi}{6}$	M1 A1 (3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2 \theta$ $6\sin^2 \theta + 10\sin \theta - 3 = 0$ $\sin \theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$ $\theta = 15.0^\circ, 165^\circ$	M1 A1 M1 A1 (4)
		(7 marks)

1	$3 \tan^2 \theta + 7 \sec \theta - 3 = 0 \Rightarrow 3(\sec^2 \theta - 1) + 7 \sec \theta - 3 = 0$	M1
	$3 \sec^2 \theta + 7 \sec \theta - 6 = 0$	A1
	$(3 \sec \theta - 2)(\sec \theta + 3) = 0 \Rightarrow \sec \theta = \dots \Rightarrow \cos \theta = \dots$	dM1
	$\theta = 109.5^\circ, 250.5^\circ$	A1, A1
		(5)

$$f(x) = 8\sin x \cos x + 4\cos^2 x - 3$$

4(a)

States or uses $\sin 2x = 2\sin x \cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$

Uses $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$

$$(f(x) =) 8\sin x \cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$$

M1

dM1

A1

(3)

(b)

$$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots \text{ ("awrt 0.464")}$$

$$(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$$

B1ft

M1

A1

(3)

(c)

(i) Maximum value = " $2\sqrt{5} - 1$ "

(ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$

$$(x =) \text{awrt } 3.69 \text{ (or } (x =) \text{awrt } 3.70)$$

B1 ft

M1

A1

(3)

(9 marks)