8(a)	Starting with the LHS: $2\operatorname{cosec}^2 2\theta (1 - \cos 2\theta) = \dfrac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \dfrac{2 - 2(1 - 2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$= \sec^2\theta = 1 + \tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = \ldots$	M1
	$\cos x = \dfrac{1}{4}$ (ignore -1)	A1
	$\cos x = \dfrac{1}{4} \Rightarrow x = \ldots$	dM1
	$x = 75.5°, 284.5°$	A1
		(4)
		(8 marks)

1	$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\left(\sec^2\theta - 1\right) + 7\sec\theta - 3 = 0$	M1
	$3\sec^2\theta + 7\sec\theta - 6 = 0$	A1
	$(3\sec\theta - 2)(\sec\theta + 3) = 0 \Rightarrow \sec\theta = \ldots \Rightarrow \cos\theta = \ldots$	dM1
	$\theta = 109.5°, 250.5°$	A1, A1
		(5)

Question Number	Scheme	Marks
4(a)	$R = \sqrt{12}$ or $2\sqrt{3}$	B1
	$\alpha = \tan^{-1}\left(\dfrac{3}{\sqrt{3}}\right) = \ldots$	M1
	$(f(x) =)\sqrt{12}\sin\left(2x - \dfrac{\pi}{3}\right)$	A1
		(3)
(b)(i) **(ii)**	Minimum value $= \dfrac{18}{\sqrt{12} + 4\sqrt{3}}\left(=\sqrt{3}\right)$	B1
	$\sin\left(6x - \dfrac{\pi}{3}\right) = 1 \Rightarrow x = \dfrac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

2(a)	$R = 25$	B1
	$\tan\alpha = \dfrac{24}{7} \Rightarrow \alpha = \ldots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \dfrac{5}{90 - 3 \times "25" \times (-1)}$	M1
	$= \dfrac{1}{33}$	A1
(b)(ii)	$(2x + "1.287") = \pi, \ldots \Rightarrow x = \ldots$	M1
	$\Rightarrow x = \dfrac{\pi - "1.287"}{2} = 0.927$	A1
		(4)
		Total 7

5 (i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \ldots$	M1
	$x = \dfrac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \dfrac{-5 \pm \sqrt{43}}{6} \left(= -1.926\ldots, 0.2595\ldots\right) \Rightarrow \theta = \arcsin(\ldots)$	M1
	$\theta = 15.0°, \ 165°$	A1
		(4)
		(7 marks)

7 (a)	$\sin 4\theta \equiv 2\sin 2\theta \cos 2\theta$	M1
	$\equiv 2\left(2\sin\theta\cos\theta\right)\left(1 - 2\sin^2\theta\right)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta\left(4 - 8\sin^2\theta\right)$	A1
		(3)
(b)	$\sec x \sin 4x = 5\sin^3 x \cot x$	
	$\dfrac{1}{\cos x} \times \cos x \sin x\left(4 - 8\sin^2 x\right) = 5\sin^3 x \cot x$	B1
	$\div \sin x \Rightarrow \ 4 - 8\sin^2 x = 5\sin^2 x \cot x$	
	$\div \cos^2 x \Rightarrow \ 4\sec^2 x - 8\tan^2 x = 5\tan^2 x \cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0 \qquad *$	A1*
		(3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \dfrac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \ldots$	dM1
	$\Rightarrow x = \text{awrt } 0.506, \ 2.08$	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = \ldots$	M1
	$x = $ awrt $5.8°$, awrt $84.2°$	A1A1
		(3)
		(9 marks)

$$f(x) = 8\sin x\cos x + 4\cos^2 x - 3$$

4(a)	States or uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	M1
	Uses $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$	dM1
	$(f(x) =) 8\sin x\cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha = \dfrac{b}{a} \Rightarrow \alpha = \ldots \ (= "\text{awrt } 0.464")$	M1
	$(f(x) =) 2\sqrt{5}\sin(2x + 0.464) - 1$	A1
		(3)
(c)	(i) Maximum value $= "2\sqrt{5} - 1"$	B1 ft
	(ii) Solves $2x + \alpha = \dfrac{5\pi}{2} \Rightarrow x = \ldots$	M1
	$(x =)$ awrt 3.69 (or $(x =)$ awrt 3.70)	A1
		(3)
		(9 marks)

5(a)	$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1
	$= (2\sin x \cos x)\cos x + (1 - 2\sin^2 x)\sin x$	dM1
	$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)

(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2(3\sin\theta - 4\sin^3\theta) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1 - \cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6 - 8(1 - \cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \dfrac{5 - \sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \dfrac{5 - \sqrt{41}}{8} \Rightarrow \theta = $ awrt $100°$ or awrt $260°$	A1
	All of $180°, 360°$, awrt $100.1°$, awrt $259.9°$	A1
		(4)
		Total 8

7 (a)	$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$	
	$\sqrt{2}(\sin x \cos 45° + \cos x \sin 45°) = \cos x \cos 60° + \sin x \sin 60°$	M1 A1
	$\sin x + \cos x = \dfrac{1}{2}\cos x + \dfrac{\sqrt{3}}{2}\sin x$	
	$\cos x = (\sqrt{3} - 2)\sin x$	M1
	$\tan x \left(= \dfrac{1}{\sqrt{3} - 2} = \dfrac{\sqrt{3} + 2}{-1} \right) = -2 - \sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x + 45° = 2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta - 45°) = -2 - \sqrt{3} \Rightarrow 2\theta - 45° = 105°, 285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta = 75°, 165°$	A1
		(4)

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv \dfrac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \dfrac{\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)}{\sin^2 x \cos^2 x} \equiv \dfrac{\cos 2x}{...}$ or $\dfrac{...}{\left(\dfrac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \dfrac{\cos 2x}{\left(\dfrac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\dfrac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x \operatorname{cosec} 2x *$	A1*
		(4)
(b)	$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta \Rightarrow \cot^2 \theta - \tan^2 \theta = 2\tan^2 \theta \Rightarrow \cot^2 \theta - 3\tan^2 \theta = 0$	M1
	$\cot^2 \theta - 3\tan^2 \theta = 0 \Rightarrow \dfrac{1}{\tan^2 \theta} - 3\tan^2 \theta = 0 \Rightarrow \tan^4 \theta = \dfrac{1}{3}$	A1
	$\tan^4 \theta = "\dfrac{1}{3}" \Rightarrow \tan \theta = \pm\sqrt[4]{"\dfrac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = ...$	M1
	$\theta = \text{awrt } 0.65, \; -0.65$	A1A1
		(5)
		Total 9