

4 (a)

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(i)

Any correct value of $A = 4$, $B = 2$ or $C = -13$

B1

Full method to find values for A , B and C

E.g. $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$

Or $x^2 + 0x + 4 \overline{) 4x^3 + 2x^2 + 3x + 8}$

M1

$$\underline{-13x}$$

$$A = 4, B = 2 \text{ and } C = -13$$

A1

(ii)

E.g. Using $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$

For example $x^2 \Rightarrow B = 2$ $x = 0 \Rightarrow 4 \times 2 + D = 8$ $D = 0$ *

B1*

(4)

(b)

$$\int \left(Ax + B + \frac{Cx}{x^2 + 4} \right) dx = \frac{Ax^2}{2} + Bx + \frac{1}{2} C \ln(k(x^2 + 4)) \quad k \text{ any constant}$$

B1 ft, M1 A1ft

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x - 8}{x^2 + 4} dx = \left[2x^2 + 2x - \frac{13}{2} \ln(k(x^2 + 4)) \right]_1^4$$

$$= \left(32 + 8 - \frac{13}{2} \ln 20 \right) - \left(2 + 2 - \frac{13}{2} \ln 5 \right) = 36 + \frac{13}{2} \ln \frac{1}{4} = 36 - 13 \ln 2$$

dM1 A1

(5)

(9 marks)

4(a)	Any correct constant, so for $A = 2$ or $B = 3$ or $C = -1$ or $D = 5$	B1
	$2x^4 + 15x^3 + 35x^2 + 21x - 4 = Ax^2(x+3)^2 + Bx(x+3)^2 + C(x+3)^2 + D$ $\Rightarrow A = \dots, B = \dots, C = \dots, D = \dots$ or $2x^4 + 15x^3 + 35x^2 + 21x - 4 \div (x^2 + 6x + 9) = \dots x^2 + \dots x + \dots + \frac{\dots}{(x+3)^2}$	M1
	2 correct of $A = 2, B = 3, C = -1, D = 5$	A1
	$A = 2, B = 3, C = -1, D = 5$	A1
		(4)
(b)	$\int f(x) dx = \int \left(2x^2 + 3x - 1 + \frac{5}{(x+3)^2} \right) dx = \frac{2x^3}{3} + \frac{3x^2}{2} - x - \frac{5}{x+3} (+c)$	M1A1ftA1
		(3)
		Total 7

2	$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$	
(a)	Sets $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$ with an attempt at one constant Attempts all 3 constants. E.g. Sets $x = 2 \Rightarrow C = \dots, -2B + C = 8 \Rightarrow B = \dots$ and compares x^2 terms gives $A = \dots$ $2x - 1 + \frac{6}{x - 2}$	M1 dM1 A1 (3)
(b)	$\int 2x - 1 + \frac{6}{x - 2} dx = x^2 - x + 6 \ln(x - 2)$ $\int_4^8 2x - 1 + \frac{6}{x - 2} dx = \left[x^2 - x + 6 \ln(x - 2) \right]_4^8 = 56 + 6 \ln 6 - 12 - 6 \ln 2$ $= 44 + 6 \ln 3$	M1 A1ft dM1 A1 (4) (7 marks)

9 (a)	$f'(x) = \frac{(2x+1) \times (12x+4) - 2(6x^2 + 4x - 2)}{(2x+1)^2}$	M1 A1
	$= \frac{12x^2 + 12x + 8}{(2x+1)^2} \text{ o.e.}$	A1
		(3)
(b)	At $x = 2 \Rightarrow f'(x) = \frac{12 \times 2^2 + 12 \times 2 + 8}{(2 \times 2 + 1)^2} = \left(\frac{16}{5} \right)$	M1
	Full method of normal $y - 6 = -\frac{5}{16}(x - 2)$	dM1
	$16y - 96 = -5x + 10 \Rightarrow 16y + 5x = 106 \quad *$	A1*
		(3)
(c)	Any correct value $A = 3, B = \frac{1}{2} \text{ or } D = -\frac{5}{2}$	B1
	$6x^2 + 4x - 2 = (Ax + B)(2x + 1) + D \Rightarrow \text{Values of } A, B \text{ and } D$	M1
	$3x + \frac{1}{2} + \frac{-5/2}{2x+1} \text{ o.e.}$	A1
		(3)
(d)	$\int 3x + \frac{1}{2} + \frac{-5/2}{2x+1} dx = \frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4} \ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4} \ln(2x+1) \right]_{\frac{1}{3}}^2 = \left(\frac{3}{2} \times 2^2 + \frac{1}{2} \times 2 - \frac{5}{4} \ln(5) \right) - \left(\frac{3}{2} \times \left(\frac{1}{3} \right)^2 + \frac{1}{2} \times \left(\frac{1}{3} \right) - \frac{5}{4} \ln\left(\frac{5}{3} \right) \right)$	dM1
	Area of R = $\int_{\frac{1}{3}}^2 3x + \frac{1}{2} + \frac{-5/2}{2x+1} dx + \frac{1}{2} \times 6 \times \left(\frac{106}{5} - 2 \right)$	M1
	$\text{Area of R} = \frac{964}{15} - \frac{5}{4} \ln 3$	A1
		(5)
		Total 14

3(a)	$(\cos 2A \equiv) \cos A \cos A - \sin A \sin A \equiv \cos^2 A - (1 - \cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2 \cos^2 A - 1$ *	A1*
		(2)
(b)	$\int (3 - 2 \cos 6x) dx = 3x - \frac{\sin 6x}{3} (+c)$	M1A1
	$\left[3x - \frac{\sin 6x}{3} \right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3 \left(\frac{\pi}{8} \right) - \frac{\sin \left(6 \times \frac{\pi}{8} \right)}{3} \right) - \left(3 \left(\frac{\pi}{12} \right) - \frac{\sin \left(6 \times \frac{\pi}{12} \right)}{3} \right) = \frac{1}{8}\pi + \frac{2 - \sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)

Question Number	Scheme	Notes
8	$\int (2 \cos x - \sin x)^2 dx = \int (4 \cos^2 x - 4 \sin x \cos x + \sin^2 x) dx$	M1
	$\int 4 \sin x \cos x dx = \int 2 \sin 2x dx = -\cos 2x$ or $\int 4 \sin x \cos x dx = -2 \cos^2 x$ or $2 \sin^2 x$	M1
	$\int (4 \cos^2 x + \sin^2 x) dx = \int (1 + 3 \cos^2 x) dx = \int \left(1 + 3 \left(\frac{\cos 2x + 1}{2} \right) \right) dx$ or $\int (4 \cos^2 x + \sin^2 x) dx = \int \left(4 \left(\frac{\cos 2x + 1}{2} \right) + \frac{1 - \cos 2x}{2} \right) dx$	M1
	$\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x + \cos 2x + \frac{5}{2} x (+c)$ or $\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x + 2 \cos^2 x + \frac{5}{2} x (+c)$ or $\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x - 2 \sin^2 x + \frac{5}{2} x (+c)$	A1A1
		(5)
		Total 5

Question Number	Scheme	Marks
6 (a)	$y = (4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} = \right) 2(4x-7)^{-\frac{1}{2}} \quad (\text{see notes})$ At $(8, 5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} \left(= \frac{2}{5} \right)$ Equation for l is $y - 5 = -\frac{5}{2}(x - 8)$ $2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0 *$	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right] \quad (\text{see notes})$ Complete area = $\int_{\frac{7}{4}}^8 (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

5(i)	$\cos 6x = 1 - 2\sin^2 3x \Rightarrow \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$ $\int \sin^2 3x dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6x \right) dx$	M1
	$= \frac{1}{2}x - \frac{1}{12}\sin 6x (+c)$	A1
		(2)
(ii)	$\int x(x^2+4)^{\frac{3}{2}} dx = \frac{1}{5}(x^2+4)^{\frac{5}{2}} (+c)$	M1A1
		(2)
		Total 4

3 (i)	$\frac{d}{dx} \ln(\sin^2 3x) = \frac{1}{\sin^2 3x} \times 2\sin 3x \times 3 \cos 3x = 6 \cot 3x$	M1 A1 (2)
(ii) (a)	$\frac{d}{dx} (3x^2 - 4)^6 = 36x(3x^2 - 4)^5$	M1 A1 (2)
(b)	$\int x(3x^2 - 4)^5 dx = \frac{1}{36}(3x^2 - 4)^6$ $\int_0^{\sqrt{2}} x(3x^2 - 4)^5 dx = \left[\frac{1}{36}(3x^2 - 4)^6 \right]_0^{\sqrt{2}} = \frac{1}{36}(2)^6 - \frac{1}{36}(-4)^6 = -112$	B1ft M1 A1cso (3) (7 marks)