7 (a)	$\mathrm{f}(x)=x^3\sqrt{4x+7} \Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3(4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+\dfrac{2x^3}{\sqrt{4x+7}}=\dfrac{3x^2(4x+7)+2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x=-\dfrac{3}{2}$ into $x^3\sqrt{4x+7} \Rightarrow y=...$	M1
	$\left(-\dfrac{3}{2},-\dfrac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4\times$"$-\dfrac{27}{8}$"	M1
	$y \leqslant \dfrac{27}{2}$	A1
		(2)
(d)	$(2,7)$	M1, A1
		(2)
		Total 10

9 (a)	$(k=)\ 4\sin^2\left(\dfrac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\dfrac{\pm x\pm 1}{4}$ or $\cos^2 y=\pm 1\pm\dfrac{x+1}{4}$ (both for dM1)	M1 dM1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{8\sin y\cos y}=\dfrac{1}{8\times\sqrt{\dfrac{x+1}{4}}\times\sqrt{\pm 1\pm\dfrac{x+1}{4}}}=\dfrac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve $=\dfrac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of $y-\dfrac{\pi}{3}=-2\sqrt{3}(x-2) \Rightarrow x=2+\dfrac{\pi}{6\sqrt{3}}$ $(=2.3...)$	dM1
	Area $=\dfrac{1}{2}\times\left(2+\dfrac{\pi}{6\sqrt{3}}\right)\times\dfrac{\pi}{3}=\dfrac{\pi}{3}+\dfrac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
7(a)	$y=\dfrac{16}{9(3x-k)}=\dfrac{16}{27x-9k}\Rightarrow\dfrac{dy}{dx}=-\dfrac{432}{(27x-9k)^2}$	M1A0
(b)	$-\dfrac{432}{(27x-9k)^2}=-12\Rightarrow 12(27x-9k)^2=432$	M1
	$12(27x-9k)^2=432\Rightarrow(27x-9k)^2=36\Rightarrow 27-9k=\pm6\Rightarrow k=\ldots$	**d**M1
	$k=\dfrac{7}{3},\dfrac{11}{3}$	A1
(c)	$y=\dfrac{16}{27-21}$	M1 (B1 on ePEN)
	$y-"\dfrac{8}{3}"=\dfrac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
(d)	$\displaystyle\int\dfrac{16}{27x-9k}\,dx=\dfrac{16}{27}[\ln(27x-9k)]$	M1
	$=\dfrac{16}{27}[\ln(27x-21)]$	A1ft
	$\dfrac{16}{27}[\ln(27x-21)]_1^3=\dfrac{16}{27}(\ln(27(3)-21)-\ln(27-21))$	**d**M1
	$=\dfrac{16}{27}\ln(10)$	A1

10 (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{dx}{dy}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{dx}{dy}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

2 (a)	$x=2y^2+5y-6$	
	$\dfrac{dx}{dy}=4y+5\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y+5=0\Rightarrow y=-\dfrac{5}{4}$	M1
	Substitutes their $y=-\dfrac{5}{4}$ into $x=2y^2+5y-6$ to find x	dM1
	$\left(-\dfrac{73}{8},-\dfrac{5}{4}\right)$	A1
		(3)

8. (a)	$f'(x)=6(2x+1)^2e^{-4x}-4(2x+1)^3e^{-4x}$	M1 A1
	$=2(2x+1)^2e^{-4x}\{3-2(2x+1)\}$	dM1
	$=2(2x+1)^2(1-4x)e^{-4x}$	A1
		(4)
(b)	Sets $f'(x)=0\Rightarrow x=-\frac{1}{2},\frac{1}{4}$	B1
	Either $f\left("-\frac{1}{2}"\right)=...$ or $f\left("\frac{1}{4}"\right)=...$	M1
	Both $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$	A1
		(3)
(c)	$\left(\frac{9}{4},\frac{27}{e}\right)$	B1ft B1ft
		(2)
		9 marks

10(a)	$\frac{1}{4}=\sin^2 4y\Rightarrow y=\frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{dx}{dy}=8\sin 4y\cos 4y$	M1A1
		(2)
(c)	$\frac{dx}{dy}=8\sin 4y\cos 4y\to\frac{dy}{dx}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{dy}{dx}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x=\frac{1}{2}$	B1ft
(ii)	$\frac{dy}{dx}=\frac{1}{4}$	B1ft
		(2)
		(9 marks)

5.(a)	$\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=\frac{\pi}{3}} = -10\sin\left(\frac{2\pi}{3}\right) - 24\cos\left(\frac{2\pi}{3}\right) = -5\sqrt{3}+12$	A1
		(3)
(b)	$R = 13$	B1
	$\tan\alpha = \frac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	M1A1
		(3)
(c)	Sets $-26\sin(2x+1.176) = 6$	M1
	$\sin(2x+\text{"}1.176\text{"}) = -\frac{3}{13}$	A1 ft
	$x = \dfrac{\arcsin\left(-\frac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	dM1, A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
6 (a)	$y = (4x-7)^{\frac{1}{2}} \quad \Rightarrow \left(\frac{dy}{dx} =\right) 2(4x-7)^{-\frac{1}{2}}$ (see notes)	M1 A1
	At $(8,5)$ gradient of tangent is $2(4\times 8-7)^{-\frac{1}{2}}\left(=\frac{2}{5}\right)$	dM1
	Equation for l is $y-5 = -\frac{5}{2}(x-8)$	ddM1
	$2y-10 = -5x+40 \Rightarrow 5x+2y-50 = 0$ *	A1*
		(5)
(b)	$\int (4x-7)^{\frac{1}{2}}\, dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]$ (see notes)	M1, A1
	Complete area $= \int_{\frac{7}{4}}^{8} (4x-7)^{\frac{1}{2}}\, dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]_{\frac{7}{4}}^{8} + 5$	dM1
	$= \frac{155}{6}$	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x) = \frac{3}{\sqrt{x}}\ln(4x) + 6\sqrt{x} \times \frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x) + 6\sqrt{x} \times \frac{1}{x} = 0 \Rightarrow \ln(4x) = -2$	dM1
	$Q\left(\frac{1}{4e^2}, -\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2 \times y$ co-ordinate of Q	M1
	Range $g(x) \leqslant \frac{12}{e}$	A1ft
		(2)
		(8 marks)

3.	$\frac{dy}{dx} = \frac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2 - 2(4x+1)(x+3) = 0$ $\Rightarrow (x+3)(10-4x) = 0 \Rightarrow x = ...$	M1
	Critical value of $\frac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\frac{dy}{dx} > 0 \Rightarrow -3 < x < \frac{5}{2}$	A1
		(6 marks)

4(a)	$\frac{49x}{x^2+x-12} + \frac{7x}{x+4} = \frac{49x + 7x(x-3)}{(x+4)(x-3)}$ or e.g. $\frac{49x(x+4)}{(x^2+x-12)(x+4)} + \frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)} = \frac{49x(x+4) + 7x(x^2+x-12)}{(x+4)(x^2+x-12)}$	M1A1
	$\frac{49x+7x(x-3)}{(x+4)(x-3)} = \frac{49x+7x^2-21x}{(x+4)(x-3)} = \frac{7x^2+28x}{(x+4)(x-3)}$ $= \frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} = \frac{7x}{(x-3)}$ *	A1*
		(3)
(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

5(a)	$$\frac{dy}{dx}=\frac{(x^2+k)\times\dfrac{2x}{x^2+k}-2x\ln(x^2+k)}{(x^2+k)^2}$$	M1
	$$\frac{dy}{dx}=\frac{2x-2x\ln(x^2+k)}{(x^2+k)^2}=\frac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$$	M1A1
		(3)
(b)	$x=0$	B1
	$"1"-\ln(x^2+k)=0\Rightarrow x^2=e^{"1"}\pm k$	M1
	$x=\pm\sqrt{e-k}$	A1ft
		(3)
(c)	Upper limit is e or $k<e$	B1ft
		(1)
		(7 marks)

Question Number	Scheme	Marks
6(a)	$\left(\frac{3\pi}{2},0\right)$	B1
		(1)
(b)	$f'(x)=\left(6\cos^2 x\right)e^{3\sin x}-\left(2\sin x\right)e^{3\sin x}$	M1A1
	$f'(x)=\left(6\cos^2 x\right)e^{3\sin x}-\left(2\sin x\right)e^{3\sin x}$ $\Rightarrow\left(6\left(1-\sin^2 x\right)-2\sin x\right)(=0)$	**d**M1
	$\Rightarrow 3\sin^2 x+\sin x-3=0$ oe	A1
		(4)
(c)	$\sin x=\frac{-1+\sqrt{37}}{6}\Rightarrow x=...$	M1
	$x=2.131$	A1
		(2)
		(7 marks)

9(a)	$x=0\Rightarrow\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)=0\Rightarrow 3y+\frac{\pi}{4}=\pi,\ 2\pi,...\Rightarrow y=...$	M1
	$y=\frac{\pi}{4}$ or $y=\frac{7\pi}{12}$	A1
	$y=\frac{\pi}{4}$ and $y=\frac{7\pi}{12}$	A1
		(3)

(b)	$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow\frac{dx}{dy}=\frac{2}{3}\times3\cos\left(3y+\frac{\pi}{4}\right)\left(=2\cos\left(3y+\frac{\pi}{4}\right)\right)$	B1
	$\Rightarrow\frac{dy}{dx}=\frac{1}{2\cos\left(3y+\frac{\pi}{4}\right)}\Rightarrow\left(\frac{dy}{dx}\right)^2=\frac{1}{4\cos^2\left(3y+\frac{\pi}{4}\right)}=\frac{1}{4\left(1-\sin^2\left(3y+\frac{\pi}{4}\right)\right)}$	M1
	$=\frac{1}{4\left(1-\left(\frac{3x}{2}\right)^2\right)}$	**d**M1
	$=\frac{1}{4-9x^2}$	A1
		(4)

(c)	Identifies that one of the gradients is $\sqrt{"4"}$ or that one of the gradients is $\dfrac{1}{\sqrt{"4"}}$ **or** Identifies that one of the gradients is 2 or that one of the gradients is $\dfrac{1}{2}$	B1ft
	At A: $y-"\dfrac{\pi}{4}"="2"x$ **and** At B: $y-"\dfrac{7\pi}{12}"="\dfrac{1}{2}"x$	M1
	$"2"x+"\dfrac{\pi}{4}"="\dfrac{1}{2}"x+"\dfrac{7\pi}{12}"\Rightarrow x=\ldots$	dM1
	$x=\dfrac{2\pi}{9}$	A1
		(4)
		Total 11

6. (a)	$f'(x)=6(2x-3)^2 e^{4x-2}+4(2x-3)^3 e^{4x-2}$	M1 A1
	$=2(2x-3)^2 e^{4x-2}\{3+2(2x-3)\}=2(4x-3)(2x-3)^2 e^{4x-2}$	dM1 A1
		(4)
(b) (i)	$x=\dfrac{3}{2},\dfrac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\dfrac{3}{4}\right)=\left(-\dfrac{3}{2}\right)^3 e^1=-\dfrac{27}{8}e,\Rightarrow g\left(\dfrac{3}{4}\right)=-27e$	M1, A1
	$-27e$ „ $g(x)$ „ 0g g ç	A1
		(4) **(8 marks)**

7(a)	$f(0)=(0-3)^2=9$	M1
	$0\leqslant f(x)\leqslant 9$	A1
		(2)
(b)	$f'(x)=-2xe^{-x^2}(2x^2-3)^2+e^{-x^2}\times 8x(2x^2-3)$	M1A1
	$=2x(2x^2-3)e^{-x^2}\left(-(2x^2-3)+4\right)=2xe^{-x^2}(2x^2-3)\left(7-2x^2\right)$	dM1A1
		(4)
(c)	$x^2=\dfrac{3}{2},\dfrac{7}{2}\Rightarrow f\left(\sqrt{\dfrac{7}{2}}\right)=e^{-\frac{7}{2}}\left(2\times\dfrac{7}{2}-3\right)^2=16e^{-\frac{7}{2}}$	M1A1
	$16e^{-\frac{7}{2}}<k<9$	dM1A1
		(4)

Question Number	Scheme	Marks
7	$\left(\frac{dy}{dx}=\right) 2e^{x^2+(3k-2)x}+2xe^{x^2+(3k-2)x}\times(2x+(3k-2))$	M1A1A1
	$\Rightarrow 2x^2+(3k-2)x+1=0 \Rightarrow (3k-2)^2-4\times2\times1$	dM1A1
	$9k^2-12k-4(>0)$	
	$9k^2-12k-4=0 \Rightarrow k=\frac{12\pm\sqrt{(-12)^2-4\times9\times(-4)}}{2\times9} \Rightarrow k=\ldots$	
	$k<\frac{2-2\sqrt{2}}{3}$ or $k>\frac{2+2\sqrt{2}}{3}$	M1A1
		(7 marks)

Question Number	Scheme	Marks
9(a)	$y=\sqrt{3+4e^{x^2}}=\left(3+4e^{x^2}\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}\times 8xe^{x^2}$	M1
	$=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\frac{\left(3+4e^{x^2}\right)}{x}=4xe^{x^2}$	dM1
	$4x^2e^{x^2}-4e^{x^2}-3=0$ *	A1*
		(3)
(c)	$f(x)=4x^2e^{x^2}-4e^{x^2}-3 \Rightarrow f(1)=-3$ AND $f(2)=652\cdot\ldots$	M1
	Change of sign and $f(x)$ is continuous hence root in $(1, 2)$	A1
		(2)
(d)	$4x^2e^{x^2}-4e^{x^2}-3=0 \Rightarrow x^2=\frac{4e^{x^2}+3}{4e^{x^2}}=\frac{4+3e^{-x^2}}{4} \Rightarrow x=\frac{1}{2}\sqrt{4+3e^{-x^2}}$ *	B1*
		(1)
(e)(i)	$x_1=1 \Rightarrow x_2=\frac{1}{2}\sqrt{4+3e^{-1}}$	M1
	$x_3=1.0997$	A1
(ii)	$\alpha=1.1051$	A1
		(3)
		Total 11

Question Number	Scheme	Marks
6(i)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{4x}{2x^2+5}$	M1A1
		(2)
(ii)	$\int \frac{21x}{3x^2+k}\mathrm{d}x=\frac{7}{2}\ln(3x^2+k)\ \ (+c)$	M1A1
	$\left[\frac{7}{2}\ln(3x^2+k)\right]_1^k<7\ln 8\Rightarrow \frac{7}{2}\ln\left(\frac{3k^2+k}{3+k}\right)<7\ln 8\Rightarrow \frac{3k^2+k}{3+k}<64$	M1
	$\frac{3k^2+k}{3+k}(<)64\Rightarrow 3k^2-63k-192\ \ (<0)\Rightarrow k=...$	dM1
	$k=23$	A1
		(5)
		(7 marks)

10.(a)	$x = 3\cos 2y \Rightarrow \left(\frac{dx}{dy} =\right) - 6\sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$	M1
	$\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$	
	$x^2 = 5 \Rightarrow a = \sqrt{5}$	M1 A1
	$\sqrt{5} = 3\cos 2y \Rightarrow b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6\sin 2y} = -\frac{1}{4}$	
	$\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2}\arcsin\left(\frac{2}{3}\right)$	M1A1
	$x = 3\cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	dM1 A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1)-2(4y^2-3)}{(2y+1)^2}$	M1 A1
	$\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2+8y+6}$	dM1, A1
		(4)
(b)	Sets $\frac{(2y+1)^2}{8y^2+8y+6} = \frac{1}{3} \Rightarrow 4y^2+4y-3=0$	M1, A1
	$\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \ldots$	dM1
	$P = \left(-1, \frac{1}{2}\right)$ only	A1
		(4)
		(8 marks)

9(a)	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}=\frac{\cos 2x}{\sin x}+\frac{2\sin x\cos x}{\cos x}$ (One Correct identity)	B1
	$=\frac{1-2\sin^2 x}{\sin x}+\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}$	M1
	$=\frac{1}{\sin x}-\frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}+2\sin x=\frac{1}{\sin x}=\text{cosec } x$ *	A1*
		(3)
(b)	E.g. Equation is $\text{cosec}^2\theta=6\cot\theta-4\Rightarrow 1+\cot^2\theta=6\cot\theta-4$	M1
	E.g. $\cot^2\theta-6\cot\theta+5=0$	A1
	E.g. $\tan\theta=\frac{1}{5},1$	dM1
	$\theta=0.197,\frac{\pi}{4}$	A1, A1
		(5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\text{cosec}x\cot x\,\text{d}x=\left[-\text{cosec}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1
	$=2-\sqrt{2}$	A1
		(2)
		10 marks