

8	(a)	$h = 1.5x - 0.5xe^{0.02x}$ $0 = 1.5d - 0.5de^{0.02d} \Rightarrow e^{0.02d} = 3$ $\Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$	M1 dM1 A1 (3)
	(b)	$\left(\frac{dh}{dx}\right) = 1.5 - \left(0.5e^{0.02x} + 0.5x \times 0.02e^{0.02x}\right)$ Sets $0 = 1.5 - 0.5e^{0.02x} - 0.5x \times 0.02e^{0.02x} \Rightarrow e^{0.02x} (0.5 + 0.5x \times 0.02) = 1.5$ $\Rightarrow e^{0.02x} = \frac{1.5}{(0.5 + 0.5x \times 0.02)}$ $\Rightarrow e^{0.02x} = \frac{1.5}{(0.5 + 0.5x \times 0.02)} = \frac{150}{50 + x} \Rightarrow x = 50 \ln \left(\frac{150}{50 + x} \right) *$	M1A1 dM1 A1* (4)
	(c)	(i) awrt 31.43 (ii) awrt 30.88 metres (including units)	M1 A1 A1 (3)
(10 marks)			

Question Number	Scheme	Marks
2(a)	$f(1) = \frac{2(1)^2 + 3(1) - 4}{e^1} - \frac{1}{1^2} = \dots \text{ and } f(2) = \frac{2(2)^2 + 3(2) - 4}{e^2} - \frac{1}{(2)^2} = \dots$ $f(1) = -0.6(321\dots) [< 0] \text{ and } f(2) = 1.(103\dots) [> 0]$ $\Rightarrow \text{There is a sign change and } f(x) \text{ is continuous over the interval hence root (in the interval } [1, 2] \text{)}$	M1 A1
		(2)
(b)	$\frac{2x^2 + 3x - 4}{e^x} - \frac{1}{x^2} = 0 \Rightarrow 2x^4 + 3x^3 = e^x + 4x^2$ $x^3(2x + 3) = e^x + 4x^2 \Rightarrow x = \sqrt[3]{\frac{e^x + 4x^2}{2x + 3}} *$	M1 A1*
		(2)
(c) (i)	$x_2 = \sqrt[3]{\frac{e^1 + 4(1)^2}{2(1) + 3}} = \text{awrt } 1.1035 \Rightarrow x_3 = \text{awrt } 1.1484$	M1A1
	(ii)	1.1813 A1
		(3)
		(7 marks)

1. (a)	$g(3) = -265, \quad g(4) = 3104$ States change of sign, continuous and hence root in $[3, 4]$	M1 A1 (2)
	$x_2 = \sqrt[3]{1000 - 2 \times 3} = 3.1591$ $(\alpha =) 3.1589$	M1 A1 A1 (3)
(5 marks)		

1. (a)	$f(x) = 2\sec x + 6x - 3$ $f(0.1) = -0.39$ $f(0.2) = 0.24$ States change of sign, continuous and hence root	M1 A1 (2)
	(b) Sets $f(x) = 0$, uses $\sec x = \frac{1}{\cos x}$ and makes x of $6x$ the subject $\Rightarrow 6x = 3 - \frac{2}{\cos x} \Rightarrow x = \frac{1}{2} - \frac{1}{3\cos x}^*$	B1* (1)
	(c) (i) $x_2 = \frac{1}{2} - \frac{1}{3\cos 0.15} = 0.16..$ $x_2 = \text{awrt } 0.1629$ (ii) $\alpha = 0.1622$	M1 A1 A1 (3)
		(6 marks)

Question Number	Scheme	Marks
2. (a)	$f(1) = 4 \times 1^3 + 2 \times 1^2 - 12 = -6$ AND $f(2) = 4 \times 2^3 + 2 \times 2^2 - 12 = 28$ States change of sign, continuous and hence root	M1 A1 (2)
(b)	$4x^3 + 2x^2 - 12 = 0 \Rightarrow x^2(4x + 2) = 12$ $\Rightarrow x^2 = \frac{12}{4x + 2} = \frac{6}{2x + 1} \Rightarrow x = \sqrt{\frac{6}{2x + 1}}$	M1 A1 (2)
(c) (i)	$\left(x_2 = \sqrt{\frac{6}{2+1}} = 1.4... \right), \quad x_3 = 1.2519$	M1, A1
(ii)	$\alpha = 1.2934$	B1 (3)
		(7 marks)

Question Number	Scheme	Marks
2(a)	$(f(2) =)2^4 - 5(2)^2 + 4(2) - 7 = -3 < 0$ and $(f(3) =)3^4 - 5(3)^2 + 4(3) - 7 = 41 > 0$	M1
	There is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> so <u>there is a root</u> (in the interval) *	A1*
		(2)
(b)	$x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}^*$	B1*
		(1)
(c)(i)	$x_2 = \sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$	M1
	= awrt 2.1179	A1
(ii)	$\alpha = \text{awrt } 2.1565$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
1(a)	$(f(1)=)1-5+e=-1.281...<0$ $(f(2)=)2^2-5\times 2+e^2=1.389...>0$	M1
	As there is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> over the interval $[1, 2]$ then <u>there is a root</u> *	A1*
		(2)
(b)(i)	$x_2 = \sqrt{5 \times 1 - e^1} = \text{awrt } 1.5105$	M1A1
(ii)	$\alpha = \text{awrt } 1.7340$	A1
		(3)
		(5 marks)

7(a)	$y = e^{-x^2} \sin 3x \Rightarrow \frac{dy}{dx} = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$	M1A1
	$3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x = 0 \Rightarrow 3 \cos 3x - 2x \sin 3x = 0$ $\Rightarrow 3 \cos 3x = 2x \sin 3x \Rightarrow \tan 3x = \frac{3}{2x}$	dM1
	$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right) *$	A1*
		(4)
(b)(i)	$x_1 = 0.4 \Rightarrow x_2 = \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4}\right)$	M1
	$(x_2 =) 0.4367$	A1
(ii)	$(x_4 =) 0.4307$	A1
		(3)

(c)	e.g. $f(x) = x - \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$ $f(0.4305) = 0.4305 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4305}\right) (= 6.38 \times 10^{-5})$ $f(0.4295) = 0.4295 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4295}\right) (= -1.141... \times 10^{-3})$ or e.g. $f(x) = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$ $f(0.4305) = 3e^{-(0.4305)^2} \cos 3(0.4305) - 2(0.4305)e^{-(0.4305)^2} \sin 3(0.4305) (= -4.968... \times 10^{-4})$ $f(0.4295) = 3e^{-(0.4295)^2} \cos 3(0.4295) - 2(0.4295)e^{-(0.4295)^2} \sin 3(0.4295) (= 8.885... \times 10^{-3})$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9