

1	$3 \tan^2 \theta + 7 \sec \theta - 3 = 0 \Rightarrow 3(\sec^2 \theta - 1) + 7 \sec \theta - 3 = 0$	M1
	$3 \sec^2 \theta + 7 \sec \theta - 6 = 0$	A1
	$(3 \sec \theta - 2)(\sec \theta + 3) = 0 \Rightarrow \sec \theta = \dots \Rightarrow \cos \theta = \dots$	dM1
	$\theta = 109.5^\circ, 250.5^\circ$	A1, A1
	(5)	
5(a)	$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1
	$= (2 \sin x \cos x) \cos x + (1 - 2 \sin^2 x) \sin x$	dM1
	$= 2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$	A1
	$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x$	A1
	(4)	
(b)	$2 \sin 3\theta = 5 \sin 2\theta \Rightarrow 2(3 \sin \theta - 4 \sin^3 \theta) = 10 \sin \theta \cos \theta$	M1
	Divides or takes out $\sin \theta$ as a factor and uses $\sin^2 \theta = 1 - \cos^2 \theta$ to set up and solve a 3TQ in $\cos \theta$ E.g. $\Rightarrow 6 \sin \theta - 8 \sin^3 \theta = 10 \sin \theta \cos \theta \Rightarrow 6 - 8(1 - \cos^2 \theta) = 10 \cos \theta$ $\Rightarrow 4 \cos^2 \theta - 5 \cos \theta - 1 = 0$ $\Rightarrow \cos \theta = \frac{5 - \sqrt{41}}{8} = (-0.175\dots)$	dM1
	Any two of the following four answers $\sin \theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ $\cos \theta = \frac{5 - \sqrt{41}}{8} \Rightarrow \theta = \text{awrt } 100^\circ \text{ or awrt } 260^\circ$	A1
	All of $180^\circ, 360^\circ, \text{awrt } 100.1^\circ, \text{awrt } 259.9^\circ$	A1
	(4)	
		Total 8

Question Number	Scheme	Marks
4(a)	$R = \sqrt{12} \text{ or } 2\sqrt{3}$ $\alpha = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \dots$ $(f(x) =) \sqrt{12} \sin\left(2x - \frac{\pi}{3}\right)$	B1 M1 A1
		(3)
(b)(i) (ii)	Minimum value = $\frac{18}{\sqrt{12} + 4\sqrt{3}} (= \sqrt{3})$ $\sin\left(6x - \frac{\pi}{3}\right) = 1 \Rightarrow x = \frac{5}{36}\pi$	B1 M1A1
		(3)
		(6 marks)

8(a)	Starting with the LHS: $2\operatorname{cosec}^2 2\theta (1 - \cos 2\theta) = \frac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \frac{2 - 2(1 - 2\sin^2 \theta)}{4\sin^2 \theta \cos^2 \theta}$	M1dM1
	$= \sec^2 \theta = 1 + \tan^2 \theta \equiv \text{RHS} \quad *$	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = \dots$	M1
	$\cos x = \frac{1}{4} \text{ (ignore } -1)$	A1
	$\cos x = \frac{1}{4} \Rightarrow x = \dots$	dM1
	$x = 75.5^\circ, 284.5^\circ$	A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
9(a)	$\frac{3\sin \theta \cos \theta}{\cos \theta + \sin \theta} = (2 + \sec 2\theta)(\cos \theta - \sin \theta)$ $\Rightarrow \frac{3}{2}\sin 2\theta = (2 + \sec 2\theta)(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)$ or $\Rightarrow 3\sin \theta \cos \theta = (2 + \sec 2\theta) \cos 2\theta$	M1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2 + \sec 2\theta) \cos 2\theta$	dM1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2 + \sec 2\theta) \cos 2\theta$	A1*
	$\Rightarrow \frac{3}{2}\sin 2\theta = 2\cos 2\theta + 1 \Rightarrow 3\sin 2\theta - 4\cos 2\theta = 2 *$	
		(3)

(b)	Way 1 using $R \sin(2x - \alpha)$ or $R \cos(2x + \alpha)$	
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots (5) \text{ or } (\alpha =) \tan^{-1}(\frac{4}{3} \text{ or } \frac{3}{4}) = \dots$	M1
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots (5) \text{ and } (\alpha =) \tan^{-1}(\frac{4}{3} \text{ or } \frac{3}{4}) = \dots$	dM1
	$3 \sin 2x - 4 \cos 2x = 2 \Rightarrow 5 \sin(2x - 0.927) = 2$ or $3 \sin 2x - 4 \cos 2x = 2 \Rightarrow 5 \cos(2x + 0.644) = -2$	A1
	$5 \sin(2x - 0.927) = 2 \Rightarrow x = \frac{\sin^{-1} \frac{2}{5} + 0.927}{2}$ or $5 \cos(2x + 0.644) = -2 \Rightarrow x = \frac{\cos^{-1} \frac{-2}{5} - 0.644}{2}$	ddM1
	$x = \text{awrt } 3.81$	A1
		(5)
		(8 marks)

4(a)	$f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$ States or uses $\sin 2x = 2 \sin x \cos x$ or $\cos 2x = \pm 2 \cos^2 x \pm 1$ Uses $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \pm 2 \cos^2 x \pm 1$ in $f(x)$ $(f(x) =) 8 \sin x \cos x + 4 \cos^2 x - 3 = 4 \sin 2x + 2 \cos 2x - 1$	M1 dM1 A1 (3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$ $\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots (= " \text{awrt } 0.464 ")$ $(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$	B1ft M1 A1 (3)
(c)	(i) Maximum value = $"2\sqrt{5} - 1"$ (ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$ $(x =) \text{awrt } 3.69 \text{ (or } (x =) \text{awrt } 3.70)$	B1 ft M1 A1 (3)
		(9 marks)

5 (i)	States $x = 2$ $\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ $x = \frac{5\pi}{6}$	B1 M1 A1 (3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2 \theta$ $6\sin^2 \theta + 10\sin \theta - 3 = 0$ $\sin \theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$ $\theta = 15.0^\circ, 165^\circ$	M1 A1 M1 A1 (4)
		(7 marks)

Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ Uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ $3 \operatorname{cosec} \theta = 8 \cos \theta \Rightarrow \sin 2\theta = \frac{3}{4}$ $\Rightarrow \theta = \frac{1}{2} \arcsin\left(\frac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	B1 M1 A1 M1 A1 (5)
(ii)	$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\frac{3}{8} \Rightarrow \tan(2x - 70^\circ) = -\frac{3}{8}$ $\arctan\left(-\frac{3}{8}\right) + 70^\circ$ Correct order of operations $x = \frac{\arctan\left(-\frac{3}{8}\right) + 70^\circ}{2}$ awrt 24.7° , awrt 114.7°	M1 A1 dM1 A1 (4)
		(9 marks)

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\dots} \text{ or } \frac{\dots}{\left(\frac{1}{2} \sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2} \sin 2x\right)^2}$	A1
	$\equiv 4 \frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4 \cot 2x \operatorname{cosec} 2x^*$	A1*
	(4)	
(b)	$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - \tan^2 \theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - 3 \tan^2 \theta = 0$	M1
	$\cot^2 \theta - 3 \tan^2 \theta = 0 \Rightarrow \frac{1}{\tan^2 \theta} - 3 \tan^2 \theta = 0 \Rightarrow \tan^4 \theta = \frac{1}{3}$	A1
	$\tan^4 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \sqrt[4]{\frac{1}{3}} = \pm 0.7598.. \Rightarrow \theta = \dots$	M1
	$\theta = \text{awrt } 0.65, -0.65$	A1A1
	(5)	
		Total 9

8(a)	$\tan 3x \equiv \tan(2x + x) \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x + x)}{\cos(2x + x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1
	$\frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \equiv \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$ $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} *$	A1*
		(3)

(b)	$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2 \sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2 \tan^2 3\theta - \tan 3\theta - 6 = 0 \quad \text{or e.g.} \Rightarrow 2 \tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2 \tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots \quad \text{or} \quad \tan 3\theta = 2 \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
		Total 8

Question Number	Scheme	Marks
9(a)	$4 \sin \theta \cos \theta = 2 \sin 2\theta$	B1
	e.g. $\Rightarrow 6 \sin^2 \theta \cot 2\theta + 2 \sin 2\theta = (3 - 3 \cos 2\theta) \frac{\cos 2\theta}{\sin 2\theta} + 2 \sin 2\theta$	M1A1 (3)
(b)	$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$	
	e.g. $\Rightarrow 3 \cot 2\theta \sin 2\theta - 14 \sin 2\theta = (3 - 3 \cos 2\theta) \cos 2\theta + 2 \sin^2 2\theta$ $\Rightarrow -14 \sin 2\theta = -3(1 - \sin^2 2\theta) + 2 \sin^2 2\theta$ $5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0 \quad *$	M1 M1 A1* (3)
(c)	$(\sin 2x =) \frac{1}{5} \Rightarrow x = \dots$ $x = \text{awrt } 5.8^\circ, \text{ awrt } 84.2^\circ$	M1 A1A1 (3)
		(9 marks)

7 (a)

$$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$$

$$\sqrt{2} (\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$$

M1 A1

$$\sin x + \cos x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$$

$$\cos x = (\sqrt{3} - 2) \sin x$$

M1

$$\tan x \left(= \frac{1}{\sqrt{3} - 2} = \frac{\sqrt{3} + 2}{-1} \right) = -2 - \sqrt{3} \quad *$$

A1*

(4)

(b)

States or uses $x + 45^\circ = 2\theta$ o.e.

B1

Proceeds from e.g. $\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta - 45^\circ = 105^\circ, 285^\circ$

M1

Correct order of operations to find one angle

dM1

$$\theta = 75^\circ, 165^\circ$$

A1

(4)

7 (a)

$$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$$

$$\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$$

$$\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$$

$$\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$$

M1

dM1

A1

(3)

(b)

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

$$\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$$

B1

$$\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$$

M1

$$\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$$

A1*

$$\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0 \quad *$$

(3)

(c)

$$\text{Uses } \sec^2 x = 1 + \tan^2 x$$

M1

$$\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$$

A1

$$\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$$

dM1

$$\Rightarrow x = \text{awrt } 0.506, 2.08$$

A1

(4)

(10 marks)

2(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90 - 3 \times 25 \times (-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x + 1.287) = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - 1.287}{2} = 0.927$	A1
		(4)
		Total 7