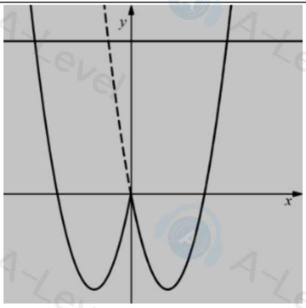
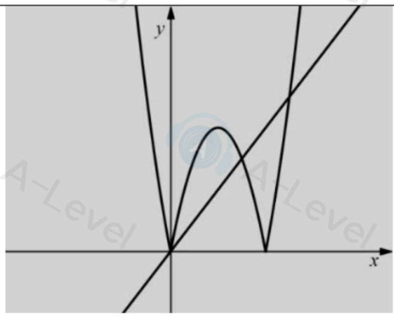
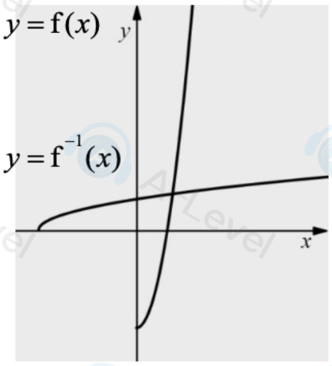
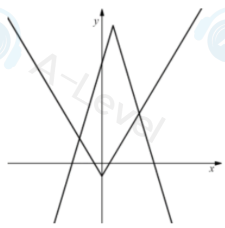
Question	Scheme	Marks
3 (a)	 $y=48$ Graphical interpretation of $f(\lvert x \rvert)=48$	
	One of $8, -8$	B1
	Attempts to solve an appropriate equation E.g. $2x^2-10x=48 \Rightarrow x^2-5x-24=0 \Rightarrow (x\pm 8)(x\pm 3)=0 \Rightarrow x=...$	M1
	$x=8, -8$ with no additional values	A1
		(3)
(b)	 Graphical interpretation of $\lvert f(x)\rvert \geqslant \frac{5}{2}x$ $y=\frac{5}{2}x$	
	Attempts to solve $2x^2-10x=\frac{5}{2}x \Rightarrow 4x^2-25x=0 \Rightarrow x=\frac{25}{4}$ OR attempts to solve (o.e.) $10x-2x^2=\frac{5}{2}x \Rightarrow 4x^2-15x=0 \Rightarrow x=\frac{15}{4}$	M1
	Attempts to solve $2x^2-10x=\frac{5}{2}x \Rightarrow 4x^2-25x=0 \Rightarrow x=\frac{25}{4}$ AND attempts to solve (o.e.) $10x-2x^2=\frac{5}{2}x \Rightarrow 4x^2-15x=0 \Rightarrow x=\frac{15}{4}$	dM1
	Achieves both critical values $x=\frac{15}{4}$, $x=\frac{25}{4}$	A1
	Correct set of values $x \leqslant \frac{15}{4}$ or $x \geqslant \frac{25}{4}$	A1
		(4)

4.(a)	$f \geqslant -5$	B1 **(1)**
(b)	 Curve starting on negative x-axis and passing through positive y-axis, in quadrants 1 and 2 only. Shape and position correct.	M1 A1 **(2)**
(c)	$2x^2-5=x$ or $2x^2-5=\sqrt{\dfrac{x+5}{2}}$ or $x=\sqrt{\dfrac{x+5}{2}}$ or $2\left(2x^2-5\right)^2-5=x$	B1
	Full attempt to solve $2x^2-x-5=0 \Rightarrow x=\ldots$ exact	M1
	$x=\dfrac{1+\sqrt{41}}{4}$	A1
		(3) **6 marks**

Question Number	Scheme	Marks
1 (a)	$(5,10)$	B1 B1
		(2)
(b)	Attempts to solve e.g. $-2(x-5)+10\ldots6x \Rightarrow x\ldots"\dfrac{5}{2}"$	M1
	$x<\dfrac{5}{2}$ o.e.	A1
		(2)
(c)	$(7,30)$	B1ft, B1ft
		(2) **(6 marks)**

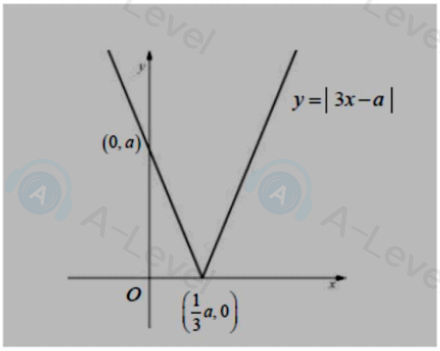
5 (a)	$f^{-1}(22) \Rightarrow 2+5\ln x=22 \Rightarrow \ln x=4 \Rightarrow (x=)\ e^4$	M1 A1
		(2)
(b)	$g(x)=\dfrac{6x-2}{2x+1} \Rightarrow g'(x)=\dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$	M1 A1
	States $\left(g'(x)=\right)\dfrac{10}{(2x+1)^2}>0$ hence increasing *	A1*
		(3)
(c)	$y=\dfrac{6x-2}{2x+1} \Rightarrow 2xy+y=6x-2 \Rightarrow 2xy-6x=-y-2$	M1
	$\Rightarrow x=\dfrac{-y-2}{2y-6}$ So $g^{-1}(x)=\dfrac{-x-2}{2x-6}$ o.e.	A1
	Domain $0<x<3$	B1
		(3)
(d)	Range fg is $fg<2+5\ln 3$	M1, A1
		(2) **(10 marks)**

Question Number	Scheme	Marks
8(a)(i)	$\left(\frac{b}{2},\ a\right)$	B1B1
(ii)	$(0,\ a-b)$	B1
(iii)	$\left(\frac{b-a}{2},\ 0\right)$ and $\left(\frac{a+b}{2},\ 0\right)$	B1B1
		(5)

(b)		B1B1
		(2)
(c)	$-x-1=2x+a-b,\ x=-3\Rightarrow 2=-6+a-b$ or $x-1=a+b-2x,\ x=5\Rightarrow 5-1=a+b-10$	M1
	$-x-1=2x+a-b,\ x=-3\Rightarrow 2=-6+a-b$ and $x-1=a+b-2x,\ x=5\Rightarrow 5-1=a+b-10$	dM1 (A1 on ePEN)
	$\begin{matrix} a-b=8 \\ a+b=14 \end{matrix}\Rightarrow a=\ldots$ or $b=\ldots$	**dd**M1
	$a=11,\ b=3$	A1
		(4)
		(11 marks)

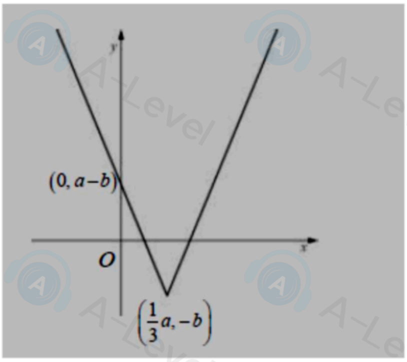
Question Number	Scheme	Marks
6(a)	(i) $P(0,\ 3a)$ (ii) $Q(a,0)$ $R\left(\frac{7}{3}a,\ 0\right)$ (iii) $S\left(\frac{5}{3}a,\ -2a\right)$	B1 B1 B1 B1
		(4)
(b)	$3x-5a-2a=x-2a\Rightarrow x=\ldots$ **or** $-(3x-5a)-2a=-(x-2a)\Rightarrow x=\ldots$	M1
	$x=\frac{5}{2}a$ **or** $x=\frac{1}{2}a$	A1
	$3x-5a-2a=x-2a\Rightarrow x=\ldots$ **and** $-(3x-5a)-2a=-(x-2a)\Rightarrow x=\ldots$	dM1
	$x=\frac{5}{2}a$ **and** $x=\frac{1}{2}a$	A1
		(4)
		Total 8

7 (a)(i)



	Shape	B1
	Vertex $\left(\frac{1}{3}a, 0\right)$	B1
	y intercept $(0, a)$	B1

(a)(ii)



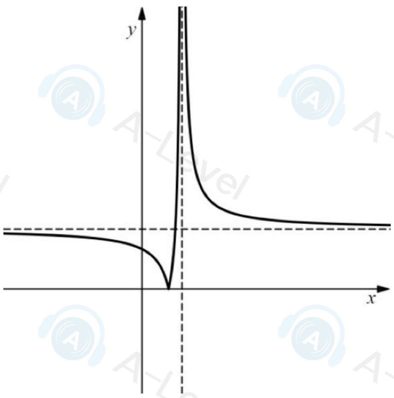
	Shape	B1ft
	Vertex $\left(\frac{1}{3}a, -b\right)$	B1ft
	y intercept $(0, a-b)$	B1ft
		(6)
(b)	Attempts to solve $-3x + a - b = 5x \Rightarrow x = \ldots$	M1
	$\Rightarrow x = \dfrac{a-b}{8}$ ONLY	A1
		(2)
		(8 marks)

(c)	$g^{-1}(x) = e^{\frac{x-3}{2}}$	M1A1
	$x \ldots 3$	B1
		(3)
(d)	$g\left(\dfrac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\dfrac{2a}{3a-5}\right) = 5$	B1
	$\ln\left(\dfrac{2a}{3a-5}\right) = 1 \Rightarrow \dfrac{2a}{3a-5} = e$	M1
	$\dfrac{2a}{3a-5} = e \Rightarrow a = \ldots$	**d**M1
	$a = \dfrac{5e}{3e-2}$	A1
		(4)

9(a)	$k=-1$	B1
		(1)
(b)(i)	$\mathrm{f}(0)=\lvert 2-4\ln(0+1)\rvert=2-0=2$	B1
(ii)	$0=2-4\ln(x+1)\Rightarrow \ln(x+1)=\dfrac{1}{2}\Rightarrow x=\mathrm{e}^{p}+q$	M1
	$x=\mathrm{e}^{\frac{1}{2}}-1$	A1
		(3)
(c)	$2-4\ln(x+1)=3\Rightarrow \ln(x+1)=\ldots$ or $-2+4\ln(x+1)=3\Rightarrow \ln(x+1)=\ldots$	M1
	$2-4\ln(x+1)=3\Rightarrow x=\ldots$ and $-2+4\ln(x+1)=3\Rightarrow x=\ldots$	dM1
	CVs $\mathrm{e}^{-\frac{1}{4}}-1,\ \mathrm{e}^{\frac{5}{4}}-1$	A1
	$"-1"<x<\mathrm{e}^{-\frac{1}{4}}-1$ or $x>\mathrm{e}^{\frac{5}{4}}-1$	ddM1A1ft
		(5)
		(9 marks)

Question Number	Scheme	Marks
1(a)	$\mathrm{gf}(1)=\mathrm{g}\left(\dfrac{2(1)}{3\times1+1}\right)=4-\left(\dfrac{2(1)}{3\times1+1}\right)^{2}$	M1
	$=\dfrac{15}{4}$ oe	A1
		(2)
(b)	$\mathrm{f}(x)\ldots 0$ or $\mathrm{f}(x)<\dfrac{2}{3}$	B1
	$0\ ,,\ \mathrm{f}(x)<\dfrac{2}{3}$	B1
		(2)

(c)	$y=\dfrac{2x}{3x+1}\Rightarrow 3xy+y=2x\Rightarrow 3xy-2x=-y\Rightarrow x(3y-2)=-y$ or $x=\dfrac{2y}{3y+1}\Rightarrow 3xy+x=2y\Rightarrow 3xy-2y=-x\Rightarrow y(3x-2)=-x$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{x}{2-3x}$	A1
		(2)
(d)	$\mathrm{f}^{-1}(x)=\mathrm{f}(x)$ **or** $\mathrm{f}^{-1}(x)=x$ **or** $\mathrm{f}(x)=x$ $"\dfrac{x}{2-3x}"=\dfrac{2x}{3x+1}$ **or** $"\dfrac{x}{2-3x}"=x$ **or** $\dfrac{2x}{3x+1}=x$ $\Rightarrow x(3x+1)=2x(2-3x)$ **or** $x=x(2-3x)$ **or** $2x=x(3x+1)$ $\Rightarrow x=\ldots$	M1
	$x=0,\ \dfrac{1}{3}$	A1
		(2)
		Total 8

8. (a)	$g(2a)=\dfrac{3\times 2a-2a}{2a-a}=4$, $\quad f(4)=\ln(2\times 4-3)=\ln 5$	M1, A1
		(2)
(b)	$y=\ln(2x-3)\Rightarrow e^{y}=2x-3\Rightarrow x=\dfrac{e^{y}+3}{2}$	M1
	$f^{-1}(x)=\dfrac{e^{x}+3}{2}\qquad x>0$	A1, B1
		(3)
(c)	 Shape of curve to right of $x=a$	B1
	Shape of curve to left of $x=a$	B1
	$y=3$ marked at correct place on curve	B1
		(3)
(d) (i)	$\dfrac{3\times 8-2a}{8-a}=10\Rightarrow a=\ldots$	M1
	$\Rightarrow a=7$	A1
(ii)	$\dfrac{3x-2\times 7}{x-7}=-10\Rightarrow x=\ldots$	dM1
	$\Rightarrow x=\dfrac{84}{13}$	A1
		(4)
		(12 marks)

6.(a)	$(2, -10)$	B1 B1
		(2)
(b)	$\text{ff}(0) = \text{f}(-4) = ...$	M1
	$= 8$	A1cso
		(2)
(c)	Attempts to solve $\quad -3(x-2)-10 = 5x+10 \Rightarrow x = ...$	M1
	$x > -\dfrac{7}{4}$ only	A1
		(2)
(d)	$x(\text{or } \lvert x\rvert) = \dfrac{16}{3}$	B1
	Attempts $3(\lvert x\rvert-2)-10 = 0 \Rightarrow \lvert x\rvert = k,\ k > 0$ or $3(-x-2)-10 = 0 \Rightarrow x = -k$ or $3(x-2)-10 = 0 \Rightarrow x = k \Rightarrow x = -k$	M1
	$x = \left(\dfrac{16}{3} \text{ and}\right) -\dfrac{16}{3}$ with no other values	A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
1(a)	$(-2, -3)$	B1
		(1)
(b)	$(-3, -9)$	B1B1
		(2)
(c)	$(-4, 3)$	B1
		(1)
		(4 marks)

10(a)(i)	$(y =)10 + k$	B1
(ii)	$x = \dfrac{10}{k}$ **or** $y = k$	B1
	$x = \dfrac{10}{k}$ **and** $y = k$	B1
		(3)
(b)	$kx - 10 + k = 2k \Rightarrow x = ...$ **or** $-kx + 10 + k = 2k \Rightarrow x = ...$	M1
	$kx - 10 + k = 2k \Rightarrow x = ...$ **and** $-kx + 10 + k = 2k \Rightarrow x = ...$	dM1
	$x \leqslant \dfrac{10-k}{k}$ or $x \geqslant \dfrac{10+k}{k}$ oe	A1
		(3)

(c)	$k > 3$	B1
	Maximum value of k occurs when $y = 3x + 1$ passes through vertex e.g. when $"k" = 3 \times "\frac{10}{k}" + 1 \Rightarrow k = ...$ **See below for alternative approaches.**	M1
	$k^2 - k - 30 = 0 \Rightarrow (k+5)(k-6) = 0 \Rightarrow k = 6$ So **maximum** k is 6	A1
	$3 < k < 6$	A1
		(4)
		Total 10

Question Number	Scheme	Marks
1 (a)	$(2, -4)$	B1, B1 **(2)**
(b)	$(8, 6)$	B1, B1 **(2)**
(c)	$(-2, 6)$	B1 **(1)**
		(5 marks)

6 (a)	$y = \frac{4x+3}{x-2} \Rightarrow yx - 2y = 4x + 3$ $\Rightarrow yx - 4x = 2y + 3$ | $y = 4 + \frac{11}{x-2} \Rightarrow y - 4 = \frac{11}{x-2}$ $x - 2 = \frac{11}{y-4}$	M1
	$\Rightarrow x = \frac{2y+3}{y-4} \Rightarrow \mathrm{f}^{-1}(x) = \frac{2x+3}{x-4}$ | $\Rightarrow x = 2 + \frac{11}{y-4} \Rightarrow \mathrm{f}^{-1}(x) = 2 + \frac{11}{x-4}$	A1
	$x \neq 4$	B1 **(3)**
(b)	$\mathrm{ff}(x) = \dfrac{4 \times \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 2}$ or $\mathrm{ff}(x) = 4 + \dfrac{11}{4 + \frac{11}{x-2} - 2}$	M1
	$= \frac{4 \times (4x+3) + 3(x-2)}{4x + 3 - 2(x-2)} = \frac{19x+6}{2x+7}$	dM1 A1 **(3)**
(c)	Either $x = 1$ or $y = 38$	M1
	$(1, 38)$	A1 **(2)**
		(8 marks)

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(x) \geqslant \ln 3$	B1
		(1)
(b)	$\mathrm{g}^{-1}(x) = \dfrac{3-2x}{x-5}$ or $-2-\dfrac{7}{x-5}$ oe $x<5$	M1A1 B1
		(3)
(c)	$\mathrm{g}(0) = \dfrac{3}{2} \Rightarrow \mathrm{f}\left(\dfrac{3}{2}\right) = \ln\left(\dfrac{21}{4}\right)$	M1A1
		(2)
(d)	$\dfrac{3+5\mathrm{e}^{2a}}{\mathrm{e}^{2a}+2} = 4 \Rightarrow 5\mathrm{e}^{2a}+3 = 4\mathrm{e}^{2a}+8 \Rightarrow \mathrm{e}^{2a} = \ldots$ $\mathrm{e}^{2a} = 5 \Rightarrow 2a = \ln 5 \Rightarrow a = \ldots$ $a = \dfrac{1}{2}\ln 5$	M1 dM1 A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
2(a)	$\mathrm{ff}(6) = \mathrm{f}\left(\text{"}\dfrac{9}{2}\text{"}\right) = \dfrac{\text{"}9/2\text{"}+3}{\text{"}9/2\text{"}-4} = \ldots;\ = 15$	M1; A1
		(2)
(b)	$\mathrm{f}^{-1}(x) = \dfrac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R},\ x \neq 1$	B1
		(3)
(c)	E.g. $\left(\dfrac{x+3}{x-4}\right)^2 + 5 = 7$ or $\dfrac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = \ldots$ or $(a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = \ldots$	dM1
	$(a=)11+7\sqrt{2}$ oe	A1
		(3)

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(x) \leqslant 9$	B1
		(1)
(b)	$\mathrm{fg}(1.5)=\mathrm{f}\left(\dfrac{3}{2\times1.5+1}\right)=9-\left(\dfrac{3}{2\times1.5+1}\right)^2$	M1
	$=\dfrac{135}{16}$	A1
		(2)
(c)	$\mathrm{g}(x)=\dfrac{3}{2x+1}\Rightarrow \mathrm{g}^{-1}(x)=\dfrac{3-x}{2x}$	M1 A1
	$0<x \leqslant 3$	B1
		(3)
		Total 6

Question Number	Scheme	Marks
8(a)	$25=a+-(5\times-2+b)\ (\Rightarrow 25=a+10-b)\Rightarrow a=15+b$ *	M1A1*
		(2)
(b)	$9=a+10+b\Rightarrow a=\ldots$ or $b=\ldots.$ $a=7,\ b=-8$	M1 A1A1
		(3)
(c)	$\left(\dfrac{8}{5},7\right)$	B1ftB1
		(2)
(d)	$15-5x=-2x^3+5x^2+4x-3\Rightarrow 2x^3-5x^2-9x+18=0$ $2x^3-5x^2-9x+18=(x+2)(2x^2-9x+9)$ $2x^2-9x+9=0\Rightarrow x=\dfrac{3}{2}$ (ignore $x=3$) $\left(\dfrac{3}{2},\dfrac{15}{2}\right)$	M1 dM1A1 ddM1 M1A1
		(6)
		(13 marks)

6 (a)	$\mathrm{gf}(2)=\mathrm{g}(3)=3^2+5$	M1
	$=14$	A1
		(2)
(b)	Correct attempt at inverse $y=6-\frac{21}{2x+3}\Rightarrow 2x+3=\frac{21}{6-y}\Rightarrow x=\left(\frac{21}{6-y}-3\right)\div 2$	M1
	$\mathrm{f}^{-1}(x)=\frac{3x+3}{2(6-x)}$ or $\mathrm{f}^{-1}(x)=\frac{21}{2(6-x)}-\frac{3}{2}$	A1
	$-1\leqslant x<6$	B1
		(3)
(c)	$\mathrm{gg}(x)=126\Rightarrow\left(x^2+5\right)^2+5=126\Rightarrow x^2+5=\sqrt{121}$	M1
	$\Rightarrow x^2="6"\Rightarrow x=(\pm)\sqrt{6}$	dM1
	$\Rightarrow x=\pm\sqrt{6}$	A1
		(3)
		Total 8