Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$ e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	B1 M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$ e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$ $\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$ $5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	M1 M1 A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = ...$ $x =$ awrt $5.8°$, awrt $84.2°$	M1 A1A1
		(3)
		(9 marks)

9 (a)	$\mathrm{f}'(x)=\dfrac{(2x+1)\times(12x+4)-2\left(6x^2+4x-2\right)}{(2x+1)^2}$	M1 A1
	$=\dfrac{12x^2+12x+8}{(2x+1)^2}$ o.e	A1
		(3)
(b)	At $x=2 \Rightarrow \mathrm{f}'(x)=\dfrac{12\times2^2+12\times2+8}{(2\times2+1)^2}=\left(\dfrac{16}{5}\right)$	M1
	Full method of normal $y-6=-\dfrac{5}{16}(x-2)$	dM1
	$16y-96=-5x+10 \Rightarrow \quad 16y+5x=106 \quad *$	A1*
		(3)
(c)	Any correct value $A=3,\ B=\dfrac{1}{2}$ *or* $D=-\dfrac{5}{2}$	B1
	$6x^2+4x-2=(Ax+B)(2x+1)+D \Rightarrow$ Values of A, B and D	M1
	$3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e	A1
		(3)
(d)	$\displaystyle\int 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\,\mathrm{d}x=\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)\right]_{\frac{1}{3}}^{2}=\left(\frac{3}{2}\times2^2+\frac{1}{2}\times2-\frac{5}{4}\ln(5)\right)-\left(\frac{3}{2}\times\left(\frac{1}{3}\right)^2+\frac{1}{2}\times\left(\frac{1}{3}\right)-\frac{5}{4}\ln\left(\frac{5}{3}\right)\right)$	dM1
	Area of $R=\displaystyle\int_{\frac{1}{3}}^{2} 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\mathrm{d}x+\frac{1}{2}\times6\times\left(\frac{106}{5}-2\right)$	M1
	Area of $R=\dfrac{964}{15}-\dfrac{5}{4}\ln 3$	A1
		(5)
		Total 14

Question Number	Scheme	Marks
9(a)	$\dfrac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta}=(2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)$ **or** $\Rightarrow 3\sin\theta\cos\theta=(2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$	**dM1**
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=2\cos 2\theta+1 \Rightarrow 3\sin 2\theta-4\cos 2\theta=2\,*$	A1*
		(3)

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **or** $(\alpha=)\tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	M1
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **and** $(\alpha=)\tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	**d**M1
	$3\sin 2x-4\cos 2x=2\Rightarrow 5\sin(2x-0.927)=2$ **or** $3\sin 2x-4\cos 2x=2\Rightarrow 5\cos(2x+0.644)=-2$	A1
	$5\sin(2x-0.927)=2\Rightarrow x=\dfrac{\sin^{-1}\frac{2}{5}+0.927}{2}$ **or** $5\cos(2x+0.644)=-2\Rightarrow x=\dfrac{\cos^{-1}\frac{-2}{5}-0.644}{2}$	**dd**M1
	$x=\text{awrt } 3.81$	A1
		(5)
		(8 marks)

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}=0\Rightarrow\ln(4x)=-2$	dM1
	$Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2\times y$ co-ordinate of Q	M1
	Range $g(x)\leqslant\frac{12}{e}$	A1ft
		(2)
		(8 marks)

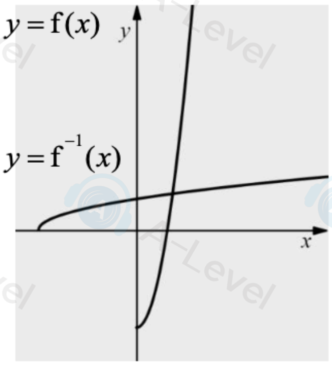
3 (i)	$\frac{d}{dx}\ln\left(\sin^2 3x\right)=\frac{1}{\sin^{\cancel{2}} 3x}\times 2\cancel{\sin 3x}\times 3\cos 3x=6\cot 3x$	M1 A1
		(2)
(ii) (a)	$\frac{d}{dx}\left(3x^2-4\right)^6=36x\left(3x^2-4\right)^5$	M1 A1
		(2)
(b)	$\int x\left(3x^2-4\right)^5 dx=\frac{1}{36}\left(3x^2-4\right)^6$	B1ft
	$\int_0^{\sqrt{2}} x\left(3x^2-4\right)^5 dx=\left[\frac{1}{36}\left(3x^2-4\right)^6\right]_0^{\sqrt{2}}=\frac{1}{36}(2)^6-\frac{1}{36}(-4)^6=-112$	M1 A1cso
		(3)
		(7 marks)

3(a)	$\log_{10} y = \pm\frac{2}{3.5}\log_{10} x - 2$	M1
	$\log_{10} y = -\frac{4}{7}\log_{10} x - 2$	A1
		(2)

(b)	$\log_{10} y = -\frac{4}{7}\log_{10} x - 2 \Rightarrow$ $y = 10^{\left(-\frac{4}{7}\log_{10} x - 2\right)}$ or $y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)}$ or $y = 10^{-\frac{4}{7}\log_{10} x} \times 10^{-2}$ or e.g. $\log_{10} y = \log_{10} x^{-\frac{4}{7}} - 2 \Rightarrow \log_{10}\frac{y}{x^{-\frac{4}{7}}} = -2$ or e.g. $\log_{10} y = -\log_{10} x^{\frac{4}{7}} - 2 \Rightarrow \log_{10} yx^{\frac{4}{7}} = -2$	M1
	$y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)}$ or $y = 10^{-\frac{4}{7}\log_{10} x} \times 10^{-2} \Rightarrow y = x^{-\frac{4}{7}} \times 10^{-2}$ or e.g. $10^{\log_{10}\frac{y}{x^{-\frac{4}{7}}}} = 10^{-2} \Rightarrow \frac{y}{x^{-\frac{4}{7}}} = 10^{-2}$ or $10^{\log_{10} yx^{\frac{4}{7}}} = 10^{-2} \Rightarrow yx^{\frac{4}{7}} = 10^{-2}$	dM1
	$y = \frac{1}{100}x^{-\frac{4}{7}}$ oe	A1
		(3)
		Total 5

Question Number	Scheme	Marks
2 (a) (i)	$\log_6 T = 4 - 2\log_6 x$	B1
(ii)	E.g. $\log_6 T = 4 - 2\log_6 216 \Rightarrow \log_6 T = 4 - 2\times 3 = -2 \Rightarrow T = \ldots$	M1
	$\Rightarrow T = 6^{-2} = \frac{1}{36}$	A1
		(3)
(b)	$\log_6 T = 4 - 2\log_6 x \Rightarrow T = 6^{4-2\log_6 x}$	M1
	$\Rightarrow T = 6^4 \times 6^{\log_6 x^{-2}}$	dM1
	$\Rightarrow T = \frac{1296}{x^2}$	A1
		(3) **(6 marks)**

9.(a)	$x=30\mathrm{e}^{-0.2\times 8}=6.06\,(\mathrm{mg})$	M1, A1
		(2)
(b)	$x=30\mathrm{e}^{-0.2\times 10}+20\mathrm{e}^{-0.2\times 2}=17.5\,(\mathrm{mg})$ *	B1*
		(1)
(c)	$30\mathrm{e}^{-0.2\times(T+8)}+20\mathrm{e}^{-0.2\times T}=10$	M1
	$\left(3\mathrm{e}^{-1.6}+2\right)\mathrm{e}^{-0.2T}=1$	A1
	$\mathrm{e}^{-0.2T}=\dfrac{1}{3\mathrm{e}^{-1.6}+2}$	
	$T=5\ln\left(3\mathrm{e}^{-1.6}+2\right)=4.79$	dM1, A1
		(4)
		(7 marks)

4.(a)	$\mathrm{f}\geqslant -5$	B1
		(1)
(b)	 Curve starting on negative x-axis and passing through positive y-axis, in quadrants 1 and 2 only.	M1
	Shape and position correct.	A1
		(2)
(c)	$2x^2-5=x$ or $2x^2-5=\sqrt{\dfrac{x+5}{2}}$ or $x=\sqrt{\dfrac{x+5}{2}}$ or $2\left(2x^2-5\right)^2-5=x$	B1
	Full attempt to solve $2x^2-x-5=0\Rightarrow x=\ldots$ exact	M1
	$x=\dfrac{1+\sqrt{41}}{4}$	A1
		(3)
		6 marks

	$f(x) = 8\sin x\cos x + 4\cos^2 x - 3$	
4(a)	States or uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	M1
	Uses $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$	dM1
	$\left(f(x) =\right) 8\sin x\cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha = \dfrac{b}{a} \Rightarrow \alpha = ...$ $\left(= \text{"awrt 0.464"}\right)$	M1
	$\left(f(x) =\right)\ 2\sqrt{5}\sin\left(2x + 0.464\right) - 1$	A1
		(3)
(c)	(i) Maximum value = $\text{"}2\sqrt{5} - 1\text{"}$	B1 ft
	(ii) Solves $2x + \alpha = \dfrac{5\pi}{2} \Rightarrow x = ...$	M1
	$\left(x =\right)$ awrt 3.69 (or $\left(x =\right)$ awrt 3.70)	A1
		(3)
		(9 marks)

5 (i)	States $x = 2$	B1	
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = ...$	M1	
	$x = \dfrac{5\pi}{6}$	A1	
			(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1	
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1	
	$\sin\theta = \dfrac{-5 \pm \sqrt{43}}{6} (= -1.926..., 0.2595...) \Rightarrow \theta = \arcsin(...)$	M1	
	$\theta = 15.0°, 165°$	A1	
			(4)
			(7 marks)

8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1 - \cos 2\theta) = \dfrac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \dfrac{2 - 2(1 - 2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$= \sec^2\theta = 1 + \tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = ...$	M1
	$\cos x = \dfrac{1}{4}$ (ignore -1)	A1
	$\cos x = \dfrac{1}{4} \Rightarrow x = ...$	dM1
	$x = 75.5°, 284.5°$	A1
		(4)
		(8 marks)

2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)

7(a)	$y = e^{-x^2}\sin 3x \Rightarrow \frac{dy}{dx} = 3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x$	M1A1
	$3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x = 0 \Rightarrow 3\cos 3x - 2x\sin 3x = 0$ $\Rightarrow 3\cos 3x = 2x\sin 3x \Rightarrow \tan 3x = \frac{3}{2x}$	dM1
	$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ *	A1*
		(4)
(b)(i)	$x_1 = 0.4 \Rightarrow x_2 = \frac{1}{3}\arctan\left(\frac{3}{2\times 0.4}\right)$	M1
	$(x_2 =)0.4367$	A1
(ii)	$(x_4 =)0.4307$	A1
		(3)

(c)	e.g. $f(x) = x - \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ $f(0.4305) = 0.4305 - \frac{1}{3}\arctan\left(\frac{3}{2\times 0.4305}\right)\left(= 6.38\times 10^{-5}\right)$ $f(0.4295) = 0.4295 - \frac{1}{3}\arctan\left(\frac{3}{2\times 0.4295}\right)\left(= -1.141...\times 10^{-3}\right)$ or e.g. $f(x) = 3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x$ $f(0.4305) = 3e^{-(0.4305)^2}\cos 3(0.4305) - 2(0.4305)e^{-(0.4305)^2}\sin 3(0.4305)\left(= -4.968...\times 10^{-4}\right)$ $f(0.4295) = 3e^{-(0.4295)^2}\cos 3(0.4295) - 2(0.4295)e^{-(0.4295)^2}\sin 3(0.4295)\left(= 8.885...\times 10^{-3}\right)$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9

9 (a)	$(k=)\ 4\sin^2\left(\frac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1 \Rightarrow \frac{dx}{dy}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\frac{\pm x\pm 1}{4}$ or $\cos^2 y=\pm 1\pm\frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx}=\frac{1}{8\sin y\cos y}=\frac{1}{8\times\sqrt{\frac{x+1}{4}}\times\sqrt{\pm 1\pm\frac{x+1}{4}}}=\frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve $=\frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of $\cancel{y}-\frac{\pi}{3}=-2\sqrt{3}(x-2) \Rightarrow x=2+\frac{\pi}{6\sqrt{3}}$ $(=2.3...)$	dM1
	Area $=\frac{1}{2}\times\left(2+\frac{\pi}{6\sqrt{3}}\right)\times\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
5(a)	$(A=)8$	B1
		(1)
(b)	$16=10+"8"e^{-Bt} \Rightarrow e^{-Bt}=...$	M1
	$e^{-45B}="\frac{3}{4}" \Rightarrow -45B=\ln"\frac{3}{4}" \Rightarrow B=...$	**dM1**
	$B=\text{awrt } 0.00639$	A1
		(3)
(c)	$\frac{dT}{dt}=-"8"\times\frac{1}{45}\ln\left(\frac{4}{3}\right)\times e^{-\frac{1}{45}\ln\left(\frac{4}{3}\right)\times 2}=...$	M1
	$=\ -0.0505$	A1
		(2)
(d)	The temperature has a (lower) limit of 10 °C **or** $5=10+"8"e^{-Bt} \Rightarrow e^{-Bt}=-\frac{5}{"8"}$ e.g. which is not possible or cannot be solved or you cannot find the log of a negative number	B1
		(1)
		(7 marks)

7 (a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$	
	$\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$	
	$\cos x=(\sqrt{3}-2)\sin x$	M1
	$\tan x\left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta=75°,165°$	A1
		(4)

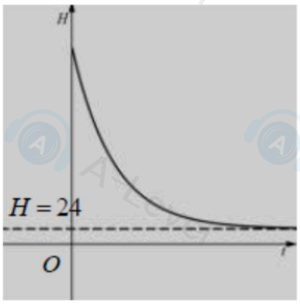
Question Number	Scheme	Marks
1 (a)	$(2,\ -4)$	B1, B1
		(2)
(b)	$(8,\ 6)$	B1, B1
		(2)
(c)	$(-2,\ 6)$	B1
		(1)
		(5 marks)

4(a)	Any correct constant, so for $A=2$ or $B=3$ or $C=-1$ or $D=5$	B1
	$2x^4+15x^3+35x^2+21x-4=Ax^2(x+3)^2+Bx(x+3)^2+C(x+3)^2+D$ $\Rightarrow A=...,B=...,C=...,D=...$ or $2x^4+15x^3+35x^2+21x-4\div(x^2+6x+9)=...x^2+...x+...+\frac{...}{(x+3)^2}$	M1
	2 correct of $A=2,B=3,C=-1,D=5$	A1
	$A=2,B=3,C=-1,D=5$	A1
		(4)
(b)	$\int f(x)\,dx=\int\left(2x^2+3x-1+\frac{5}{(x+3)^2}\right)dx=\frac{2x^3}{3}+\frac{3x^2}{2}-x-\frac{5}{x+3}(+c)$	M1A1ftA1
		(3)
		Total 7

6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$	M1 A1
	$= 2(2x-3)^2 e^{4x-2}\{3+2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	dM1 A1
		(4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$	M1, A1
	$-27e \leqslant g(x) \leqslant 0$ g g	A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \frac{5}{16}x + 1.5$	M1A1
		(2)
(b)	$\log_{10} y = "\frac{5}{16}"x + "1.5" \Rightarrow y = 10^{"\frac{5}{16}"x + "1.5"}$	M1
	$\Rightarrow y = 10^{"\frac{5}{16}"x} \times 10^{"1.5"}$	M1
	$y = 31.6 \times 2.05^x$	A1
		(3)
		Total 5

3(a)	$(\cos 2A \equiv) \cos A \cos A - \sin A \sin A \equiv \cos^2 A - (1-\cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2\cos^2 A - 1$ *	A1*
		(2)
(b)	$\int (3 - 2\cos 6x)\,dx = 3x - \frac{\sin 6x}{3} \quad (+c)$	M1A1
	$\left[3x - \frac{\sin 6x}{3}\right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3\left(\frac{\pi}{8}\right) - \frac{\sin\left(6\times\frac{\pi}{8}\right)}{3}\right) - \left(3\left(\frac{\pi}{12}\right) - \frac{\sin\left(6\times\frac{\pi}{12}\right)}{3}\right) = \frac{1}{8}\pi + \frac{2-\sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)

5 (a)	$304(°C)$	B1
		(1)
(b)	 Shape or Asymptote Fully correct Shape and Asymptote	M1 A1
		(2)
(c)	$144 = 280\mathrm{e}^{-0.05t} + 24 \quad \Rightarrow 280\mathrm{e}^{-0.05t} = 120$	M1
	Correct use of lns $\Rightarrow -0.05t = \ln\dfrac{120}{280} \Rightarrow t = 16.95$	dM1 A1
		(3)
(d)	$\left(\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right) -0.05\times 280\mathrm{e}^{-0.05t}$	M1
	$\dfrac{\mathrm{d}H}{\mathrm{d}t} = -0.05\times(H-24) \Rightarrow \dfrac{\mathrm{d}H}{\mathrm{d}t} = 1.2 - 0.05H$	M1, A1cso
		(3)
		(9 marks)

6.(a)	$(2,-10)$	B1 B1
		(2)
(b)	$\mathrm{ff}(0) = \mathrm{f}(-4) = \ldots$	M1
	$= 8$	A1cso
		(2)
(c)	Attempts to solve $\quad -3(x-2)-10 = 5x+10 \Rightarrow x = \ldots$	M1
	$x > -\dfrac{7}{4}$ only	A1
		(2)
(d)	$x(\text{or } \lvert x\rvert) = \dfrac{16}{3}$	B1
	Attempts $3(\lvert x\rvert-2)-10=0 \Rightarrow \lvert x\rvert = k,\ k>0$ or $3(-x-2)-10=0 \Rightarrow x=-k$ or $3(x-2)-10=0 \Rightarrow x=k \Rightarrow x=-k$	M1
	$x = \left(\dfrac{16}{3} \text{ and}\right) -\dfrac{16}{3}$ with no other values	A1
		(3)
		(9 marks)

7 (a)	$f(x)=x^3\sqrt{4x+7} \Rightarrow f'(x)=3x^2\sqrt{4x+7}+2x^3(4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow f'(x)=3x^2\sqrt{4x+7}+\dfrac{2x^3}{\sqrt{4x+7}}=\dfrac{3x^2(4x+7)+2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow f'(x)=\dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x=-\dfrac{3}{2}$ into $x^3\sqrt{4x+7} \Rightarrow y=...$	M1
	$\left(-\dfrac{3}{2},-\dfrac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4\times$"$-\dfrac{27}{8}$"	M1
	$y \leqslant \dfrac{27}{2}$	A1
		(2)
(d)	$(2,7)$	M1, A1
		(2)
		Total 10

Question Number	Scheme	Marks
2(a)	$ff(6)=f\left(\text{"}\dfrac{9}{2}\text{"}\right)=\dfrac{\text{"}9/2\text{"}+3}{\text{"}9/2\text{"}-4}=...;\ =15$	M1; A1
		(2)
(b)	$f^{-1}(x)=\dfrac{4x+3}{x-1}$	M1A1
	$x\in\mathbb{R},\ x\neq 1$	B1
		(3)
(c)	E.g. $\left(\dfrac{x+3}{x-4}\right)^2+5=7$ or $\dfrac{a+3}{a-4}=(\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2-22x+23=0\Rightarrow x=...$ or $(a+3)=(\pm)\sqrt{2}(a-4)\Rightarrow a=...$	dM1
	$(a=)11+7\sqrt{2}$ oe	A1
		(3)

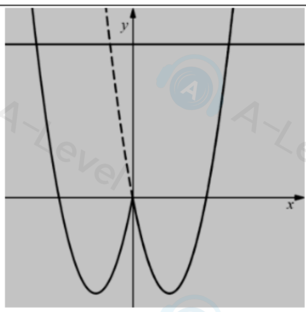
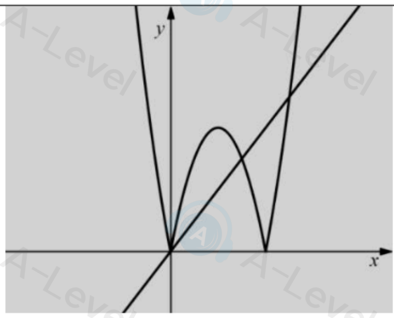
4 (a)	$\log_{10}N=2\Rightarrow N=100$	B1
		(1)
(b)	$\log_{10}N=0.35t+2\Rightarrow N=10^{0.35t+2}$	M1
	$\Rightarrow N=10^{0.35t}\times 10^2$	dM1
	$\Rightarrow N=100\times 2.24^t$	A1
		(3)
(c)	Rate of growth at 5 hours is $\dfrac{dN}{dt}=100\times\ln 2.24\times 2.24^5$	M1
	Allow an answer in the range 4530 to 4550 (bacteria per hour)	A1
		(2)
		Total 6

Question Number	Scheme	Marks
6 (a)	$y=(4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx}=\right) 2(4x-7)^{-\frac{1}{2}}$ (see notes)	M1 A1
	At $(8,5)$ gradient of tangent is $2(4\times 8-7)^{-\frac{1}{2}}\left(=\frac{2}{5}\right)$	dM1
	Equation for l is $y-5=-\frac{5}{2}(x-8)$	ddM1
	$2y-10=-5x+40 \Rightarrow 5x+2y-50=0$ *	A1*
		(5)
(b)	$\int (4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]$ (see notes)	M1, A1
	Complete area $=\int_{\frac{7}{4}}^{8}(4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]_{\frac{7}{4}}^{8}+5$	dM1
	$=\frac{155}{6}$	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
7	$\left(\frac{dy}{dx}=\right) 2e^{x^2+(3k-2)x}+2xe^{x^2+(3k-2)x}\times(2x+(3k-2))$	M1A1A1
	$\Rightarrow 2x^2+(3k-2)x+1=0 \Rightarrow (3k-2)^2-4\times 2\times 1$	dM1A1
	$9k^2-12k-4(>0)$	
	$9k^2-12k-4=0 \Rightarrow k=\frac{12\pm\sqrt{(-12)^2-4\times 9\times(-4)}}{2\times 9} \Rightarrow k=\ldots$	
	$k<\frac{2-2\sqrt{2}}{3}$ or $k>\frac{2+2\sqrt{2}}{3}$	M1A1
		(7 marks)

4(a)	$\frac{49x}{x^2+x-12}+\frac{7x}{x+4}=\frac{49x+7x(x-3)}{(x+4)(x-3)}$ or e.g. $\frac{49x(x+4)}{(x^2+x-12)(x+4)}+\frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)}=\frac{49x(x+4)+7x(x^2+x-12)}{(x+4)(x^2+x-12)}$	M1A1
	$\frac{49x+7x(x-3)}{(x+4)(x-3)}=\frac{49x+7x^2-21x}{(x+4)(x-3)}=\frac{7x^2+28x}{(x+4)(x-3)}$ $=\frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}=\frac{7x}{(x-3)}$ *	A1*
		(3)

(b)	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}\Rightarrow \mathrm{f}'(x)=\dfrac{(x-3)\times 7-7x\times 1}{(x-3)^2}$	M1
	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}\Rightarrow \mathrm{f}'(x)=-\dfrac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}=7+\dfrac{21}{x-3}\Rightarrow \mathrm{f}'(x)=-\dfrac{21}{(x-3)^2}$	M1A1
		Total 5

3 (a)	 $y=48$ Graphical interpretation of $\mathrm{f}(\lvert x\rvert)=48$	
	One of $8, -8$	B1
	Attempts to solve an appropriate equation E.g. $2x^2-10x=48\Rightarrow x^2-5x-24=0\Rightarrow (x\pm 8)(x\pm 3)=0\Rightarrow x=...$	M1
	$x=8, -8$ with no additional values	A1
		(3)
(b)	 $y=\dfrac{5}{2}x$ Graphical interpretation of $\lvert \mathrm{f}(x)\rvert \geqslant \dfrac{5}{2}x$	
	Attempts to solve $2x^2-10x=\dfrac{5}{2}x\Rightarrow 4x^2-25x=0\Rightarrow x=\dfrac{25}{4}$ OR attempts to solve (o.e.) $10x-2x^2=\dfrac{5}{2}x\Rightarrow 4x^2-15x=0\Rightarrow x=\dfrac{15}{4}$	M1
	Attempts to solve $2x^2-10x=\dfrac{5}{2}x\Rightarrow 4x^2-25x=0\Rightarrow x=\dfrac{25}{4}$ AND attempts to solve (o.e.) $10x-2x^2=\dfrac{5}{2}x\Rightarrow 4x^2-15x=0\Rightarrow x=\dfrac{15}{4}$	dM1
	Achieves both critical values $x=\dfrac{15}{4}$, $x=\dfrac{25}{4}$	A1
	Correct set of values $x\leqslant\dfrac{15}{4}$ or $x\geqslant\dfrac{25}{4}$	A1
		(4)

Question	Scheme	Marks
5(a)	$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$	M1
	$= (2\sin x\cos x)\cos x + \left(1-2\sin^2 x\right)\sin x$	dM1
	$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x\left(1-\sin^2 x\right) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2\left(3\sin\theta - 4\sin^3\theta\right) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6-8(1-\cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \dfrac{5-\sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \dfrac{5-\sqrt{41}}{8} \Rightarrow \theta =$ awrt 100° or awrt 260°	A1
	All of 180°, 360°, awrt 100.1°, awrt 259.9°	A1
		(4)
		Total 8

3.	$\dfrac{dy}{dx} = \dfrac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2 - 2(4x+1)(x+3) = 0$ $\Rightarrow (x+3)(10-4x) = 0 \Rightarrow x = ...$	M1
	Critical value of $\dfrac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\dfrac{dy}{dx} > 0 \Rightarrow -3 < x < \dfrac{5}{2}$	A1
		(6 marks)

10(a)(i)	$(y=)10+k$	B1
(ii)	$x = \dfrac{10}{k}$ **or** $y = k$	B1
	$x = \dfrac{10}{k}$ **and** $y = k$	B1
		(3)

(b)	$kx - 10 + k = 2k \Rightarrow x = ...$ **or** $-kx + 10 + k = 2k \Rightarrow x = ...$	M1
	$kx - 10 + k = 2k \Rightarrow x = ...$ **and** $-kx + 10 + k = 2k \Rightarrow x = ...$	dM1
	$x \leqslant \dfrac{10-k}{k}$ or $x \geqslant \dfrac{10+k}{k}$ oe	A1
		(3)

(c)	$k > 3$	B1
	Maximum value of k occurs when $y = 3x + 1$ passes through vertex e.g. when $"k" = 3\times"\frac{10}{k}"+1 \Rightarrow k = ...$ **See below for alternative approaches.**	M1
	$k^2 - k - 30 = 0 \Rightarrow (k+5)(k-6) = 0 \Rightarrow k = 6$ So **maximum** k is 6	A1
	$3 < k < 6$	A1
		(4)
		Total 10

9(a)	$k = -1$	B1
		(1)
(b)(i)	$f(0) = \lvert 2 - 4\ln(0+1)\rvert = 2 - 0 = 2$	B1
(ii)	$0 = 2 - 4\ln(x+1) \Rightarrow \ln(x+1) = \frac{1}{2} \Rightarrow x = e^p + q$	M1
	$x = e^{\frac{1}{2}} - 1$	A1
		(3)
(c)	$2 - 4\ln(x+1) = 3 \Rightarrow \ln(x+1) = ...$ or $-2 + 4\ln(x+1) = 3 \Rightarrow \ln(x+1) = ...$	M1
	$2 - 4\ln(x+1) = 3 \Rightarrow x = ...$ and $-2 + 4\ln(x+1) = 3 \Rightarrow x = ...$	dM1
	CVs $e^{-\frac{1}{4}} - 1,\ e^{\frac{5}{4}} - 1$	A1
	$"-1" < x < e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$	ddM1A1ft
		(5)
		(9 marks)

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2\,dx = \int (4\cos^2 x - 4\sin x\cos x + \sin^2 x)\,dx$	M1
	$\int 4\sin x\cos x\,dx = \int 2\sin 2x\,dx = -\cos 2x$ or $\int 4\sin x\cos x\,dx = -2\cos^2 x$ or $2\sin^2 x$	M1
	$\int (4\cos^2 x + \sin^2 x)\,dx = \int (1 + 3\cos^2 x)\,dx = \int \left(1 + 3\left(\frac{\cos 2x + 1}{2}\right)\right)dx$ or $\int (4\cos^2 x + \sin^2 x)\,dx = \int \left(4\left(\frac{\cos 2x + 1}{2}\right) + \frac{1 - \cos 2x}{2}\right)dx$	M1
	$\int (2\cos x - \sin x)^2\,dx = \frac{3}{4}\sin 2x + \cos 2x + \frac{5}{2}x(+c)$ or $\int (2\cos x - \sin x)^2\,dx = \frac{3}{4}\sin 2x + 2\cos^2 x + \frac{5}{2}x(+c)$ or $\int (2\cos x - \sin x)^2\,dx = \frac{3}{4}\sin 2x - 2\sin^2 x + \frac{5}{2}x(+c)$	A1A1
		(5)
		Total 5

6(a)	$\log_{10} S = 4.5 - 0.006\times 2 \Rightarrow S = 10^{4.5-0.006\times 2} = 30800\,\text{km}^2$	M1A1
		(2)
(b)	$\log_{10} S = 4.5 - 0.006t \Rightarrow S = 10^{4.5-0.006t}$ (or $p = 10^{4.5}$ or $q = 10^{-0.006}$)	M1
	$S = 10^{4.5-0.006t} = 10^{4.5}\times\left(10^{-0.006}\right)^t$ (or $p = 10^{4.5}$ and $q = 10^{-0.006}$)	dM1
	$S = 31600\times(0.986)^t$	A1
		(3)
(c)	E.g. The proportion of area covered by coral reefs retained from year to year.	B1
		(1)
		(6 marks)

9(a)	$x = 0 \Rightarrow \frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right) = 0 \Rightarrow 3y + \frac{\pi}{4} = \pi,\ 2\pi, \ldots \Rightarrow y = \ldots$	M1
	$y = \frac{\pi}{4}$ or $y = \frac{7\pi}{12}$	A1
	$y = \frac{\pi}{4}$ and $y = \frac{7\pi}{12}$	A1
		(3)

(b)	$x = \frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3}\times 3\cos\left(3y+\frac{\pi}{4}\right)\left(= 2\cos\left(3y+\frac{\pi}{4}\right)\right)$	B1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\cos\left(3y+\frac{\pi}{4}\right)} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4\cos^2\left(3y+\frac{\pi}{4}\right)} = \frac{1}{4\left(1-\sin^2\left(3y+\frac{\pi}{4}\right)\right)}$	M1
	$= \frac{1}{4\left(1-\left(\frac{3x}{2}\right)^2\right)}$	**dM1**
	$= \frac{1}{4-9x^2}$	A1
		(4)

(c)	Identifies that one of the gradients is $\sqrt{"4"}$ or that one of the gradients is $\frac{1}{\sqrt{"4"}}$ **or** Identifies that one of the gradients is 2 or that one of the gradients is $\frac{1}{2}$	B1ft
	At A: $y - "\frac{\pi}{4}" = "2"x$ **and** At B: $y - "\frac{7\pi}{12}" = "\frac{1}{2}"x$	M1
	$"2"x + "\frac{\pi}{4}" = "\frac{1}{2}"x + "\frac{7\pi}{12}" \Rightarrow x = \ldots$	**dM1**
	$x = \frac{2\pi}{9}$	A1
		(4)
		Total 11

2 (a)	$10\ \text{m}^2$	B1 **(1)**
(b)	$\log_{10} 25 = 1 + 0.03T \Rightarrow T = \ldots$ $T = 13.26$	M1 A1cso **(2)**
		(3 marks)

2(a)	$R=25$	B1
	$\tan\alpha=\frac{24}{7}\Rightarrow\alpha=...$	M1
	$\alpha=1.287$	A1
		(3)
(b)(i)	$\text{Min}=\frac{5}{90-3\times"25"\times(-1)}$	M1
	$=\frac{1}{33}$	A1
(b)(ii)	$(2x+"1.287")=\pi,...\Rightarrow x=...$	M1
	$\Rightarrow x=\frac{\pi-"1.287"}{2}=0.927$	A1
		(4)
		Total 7

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x-\tan^2 x\equiv\frac{\cos^2 x}{\sin^2 x}-\frac{\sin^2 x}{\cos^2 x}\equiv\frac{\cos^4 x-\sin^4 x}{\sin^2 x\cos^2 x}$	M1
	$\equiv\frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{\sin^2 x\cos^2 x}\equiv\frac{\cos 2x}{...}$ or $\frac{...}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv\frac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\frac{\cos 2x}{\sin 2x\sin 2x}\equiv 4\cot 2x\,\text{cosec}2x$ *	A1*
		(4)
(b)	$4\cot 2\theta\,\text{cosec}2\theta=2\tan^2\theta\Rightarrow\cot^2\theta-\tan^2\theta=2\tan^2\theta\Rightarrow\cot^2\theta-3\tan^2\theta=0$	M1
	$\cot^2\theta-3\tan^2\theta=0\Rightarrow\frac{1}{\tan^2\theta}-3\tan^2\theta=0\Rightarrow\tan^4\theta=\frac{1}{3}$	A1
	$\tan^4\theta="\frac{1}{3}"\Rightarrow\tan\theta=\pm\sqrt[4]{"\frac{1}{3}"}="\pm 0.7598.."\Rightarrow\theta=...$	M1
	$\theta=\text{awrt }0.65,\ -0.65$	A1A1
		(5)
		Total 9

9(a)	$\dfrac{\cos 2x}{\sin x}+\dfrac{\sin 2x}{\cos x}=\dfrac{\cos 2x}{\sin x}+\dfrac{2\sin x\cos x}{\cos x}$ (One Correct identity)	B1
	$=\dfrac{1-2\sin^2 x}{\sin x}+\dfrac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}$	M1
	$=\dfrac{1}{\sin x}-\dfrac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}+2\sin x=\dfrac{1}{\sin x}=\operatorname{cosec} x$ *	A1*
		(3)
(b)	E.g. Equation is $\operatorname{cosec}^2\theta=6\cot\theta-4\Rightarrow 1+\cot^2\theta=6\cot\theta-4$	M1
	E.g. $\cot^2\theta-6\cot\theta+5=0$	A1
	E.g. $\tan\theta=\dfrac{1}{5},1$	dM1
	$\theta=0.197,\dfrac{\pi}{4}$	A1, A1
		(5)
(c)	$\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\operatorname{cosec}x\cot x\,dx=\left[-\operatorname{cosec}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1
	$=2-\sqrt{2}$	A1
		(2)
		10 marks

Question Number	Scheme	Marks
2. (a)	$f(1)=4\times1^3+2\times1^2-12=-6$ AND $f(2)=4\times2^3+2\times2^2-12=28$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	$4x^3+2x^2-12=0\Rightarrow x^2(4x+2)=12$	M1
	$\Rightarrow x^2=\dfrac{12}{4x+2}=\dfrac{6}{2x+1}\Rightarrow x=\sqrt{\dfrac{6}{2x+1}}$	A1
		(2)
(c) (i)	$\left(x_2=\sqrt{\dfrac{'6'}{2+1}}=1.4...\right)$, $x_3=1.2519$	M1, A1
(ii)	$\alpha=1.2934$	B1
		(3)
		(7 marks)

10.(a)	$x = 3\cos 2y \Rightarrow \left(\frac{dx}{dy} =\right) - 6\sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$	M1
	$\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$	
	$x^2 = 5 \Rightarrow a = \sqrt{5}$	M1 A1
	$\sqrt{5} = 3\cos 2y \Rightarrow b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6\sin 2y} = -\frac{1}{4}$	
	$\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2}\arcsin\left(\frac{2}{3}\right)$	M1A1
	$x = 3\cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	dM1 A1
		(4)
		(9 marks)

8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t}$	M1, A1
	Sets $-8\,e^{-0.2t} + 18\,e^{-0.9t} = 0 \Rightarrow 4e^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln\frac{9}{4} \Rightarrow t = ...$	M1
	$T = 1.158$ (minutes)	A1
		(5)
(d)	$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8}$	M1
	$\Rightarrow e^{0.2t} = \frac{8}{1+4\,e^{-0.9t}} \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$	A1*
		(2)
(e)	$t_2 = 5\ln\left(\frac{8}{1+4e^{-0.9\times 10}}\right) = ...$	M1
	$(t_2) =$ awrt 10.3947	A1
	$(M) = 10.3955$	A 1
		(3)
		Total 12

4 (a)	$\frac{4x^3+2x^2+3x+8}{x^2+4} \equiv Ax+B+\frac{Cx+D}{x^2+4}$	
(i)	Any correct value of $A=4,\ B=2$ or $C=-13$	B1
	Full method to find values for A, B and C E.g. $4x^3+2x^2+3x+8 \equiv (Ax+B)\left(x^2+4\right)+Cx+D$ Or $\begin{array}{r} 4x+2 \\ x^2+0x+4\overline{)4x^3+2x^2+3x+8} \\ \hline -13x \end{array}$	M1
	$A=4,\ B=2$ and $C=-13$	A1
(ii)	E.g. Using $4x^3+2x^2+3x+8 \equiv (Ax+B)\left(x^2+4\right)+Cx+D$	
	For example $x^2 \Rightarrow B=2 \qquad x=0 \Rightarrow 4\times 2+D=8 \qquad D=0$ *	B1*
		(4)
(b)	$\int\left(Ax+B+\frac{Cx}{x^2+4}\right)\mathrm{d}x=\frac{Ax^2}{2}+Bx,+\frac{1}{2}C\ln\left(k\left(x^2+4\right)\right)$ k any constant	B1 ft, M1 A1ft
	$\int_1^4 \frac{4x^3+2x^2+3x-8}{x^2+4}\mathrm{d}x=\left[2x^2+2x-\frac{13}{2}\ln\left(k\left(x^2+4\right)\right)\right]_1^4$	
	$=\left(32+8-\frac{13}{2}\ln 20\right)-\left(2+2-\frac{13}{2}\ln 5\right)=36+\frac{13}{2}\ln\frac{1}{4}=36-13\ln 2$	dM1 A1
		(5)
		(9 marks)

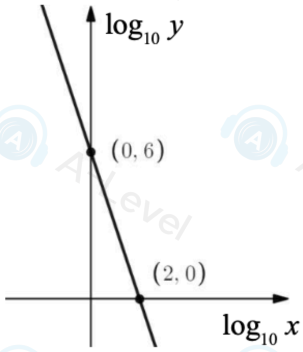
7(a)	$\mathrm{f}(0)=(0-3)^2=9$	M1
	$0 \leqslant \mathrm{f}(x) \leqslant 9$	A1
		(2)
(b)	$\mathrm{f}'(x)=-2x\mathrm{e}^{-x^2}(2x^2-3)^2+\mathrm{e}^{-x^2}\times 8x(2x^2-3)$	M1A1
	$=2x(2x^2-3)\mathrm{e}^{-x^2}\left(-(2x^2-3)+4\right)=2x\mathrm{e}^{-x^2}(2x^2-3)\left(7-2x^2\right)$	dM1A1
		(4)
(c)	$x^2=\frac{3}{2},\ \frac{7}{2} \Rightarrow \mathrm{f}\left(\sqrt{\frac{7}{2}}\right)=\mathrm{e}^{-\frac{7}{2}}\left(2\times\frac{7}{2}-3\right)^2=16\mathrm{e}^{-\frac{7}{2}}$	M1A1
	$16\mathrm{e}^{-\frac{7}{2}} < k < 9$	dM1A1
		(4)

6 (a)	$\mathrm{gf}(2)=\mathrm{g}(3)=3^2+5$	M1
	$=14$	A1
		(2)
(b)	Correct attempt at inverse $y=6-\dfrac{21}{2x+3}\Rightarrow 2x+3=\dfrac{21}{6-y}\Rightarrow x=\left(\dfrac{21}{6-y}-3\right)\div 2$	M1
	$\mathrm{f}^{-1}(x)=\dfrac{3x+3}{2(6-x)}$ or $\mathrm{f}^{-1}(x)=\dfrac{21}{2(6-x)}-\dfrac{3}{2}$	A1
	$-1 \leqslant x<6$	B1
		(3)
(c)	$\mathrm{gg}(x)=126\Rightarrow\left(x^2+5\right)^2+5=126\Rightarrow x^2+5=\sqrt{121}$	M1
	$\Rightarrow x^2="6"\Rightarrow x=(\pm)\sqrt{6}$	dM1
	$\Rightarrow x=\pm\sqrt{6}$	A1
		(3)
		Total 8

Question Number	Scheme	Marks
2(a)	$\mathrm{f}(1)=\dfrac{2(1)^2+3(1)-4}{e^1}-\dfrac{1}{1^2}=\ldots$ **and** $\mathrm{f}(2)=\dfrac{2(2)^2+3(2)-4}{e^2}-\dfrac{1}{(2)^2}=\ldots$	M1
	$\mathrm{f}(1)=-0.6(321\ldots)[<0]$ **and** $\mathrm{f}(2)=1.(103\ldots)[>0]$ $\Rightarrow$ There is a sign change and f(x) is continuous over the interval hence root (in the interval $[1,2]$)	A1
		(2)
(b)	$\dfrac{2x^2+3x-4}{e^x}-\dfrac{1}{x^2}=0\Rightarrow 2x^4+3x^3=e^x+4x^2$	M1
	$x^3(2x+3)=e^x+4x^2\Rightarrow x=\sqrt[3]{\dfrac{e^x+4x^2}{2x+3}}$ *	A1*
		(2)
(c) (i)	$x_2=\sqrt[3]{\dfrac{e^1+4(1)^2}{2(1)+3}}=\text{awrt }1.1035\Rightarrow x_3=\text{awrt }1.1484$	M1A1
(ii)	1.1813	A1
		(3)
		(7 marks)

	$f(x) = 2\sec x + 6x - 3$	
1. (a)	$f(0.1) = -0.39 \qquad f(0.2) = 0.24$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	Sets $f(x) = 0$, uses $\sec x = \dfrac{1}{\cos x}$ and makes x of $6x$ the subject	
	$\Rightarrow 6x = 3 - \dfrac{2}{\cos x} \Rightarrow x = \dfrac{1}{2} - \dfrac{1}{3\cos x}$ *	B1*
		(1)
(c)	(i) $x_2 = \dfrac{1}{2} - \dfrac{1}{3\cos 0.15} = 0.16..$	M1
	$x_2 =$ awrt 0.1629	A1
	(ii) $\alpha = 0.1622$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
6(a)	(i) $P(0,\ 3a)$	B1
	(ii) $Q(a,0)\quad R\left(\dfrac{7}{3}a,\ 0\right)$	B1 B1
	(iii) $S\left(\dfrac{5}{3}a,\ -2a\right)$	B1
		(4)
(b)	$3x-5a-2a = x-2a \Rightarrow x = ...$ **or** $-(3x-5a)-2a = -(x-2a) \Rightarrow x = ...$	M1
	$x = \dfrac{5}{2}a$ **or** $x = \dfrac{1}{2}a$	A1
	$3x-5a-2a = x-2a \Rightarrow x = ...$ **and** $-(3x-5a)-2a = -(x-2a) \Rightarrow x = ...$	dM1
	$x = \dfrac{5}{2}a$ **and** $x = \dfrac{1}{2}a$	A1
		(4)
		Total 8

3 (i)	$y=\frac{10^6}{x^3} \Rightarrow \log_{10} y = \log_{10} 10^6 - \log_{10} x^3 \Rightarrow \log_{10} y = 6 - 3\log_{10} x$ 	B1 B1 B1 **(3)**
(ii)	States $\log_3 N = 2t+4$ **or** $\log_3 N = \log_3 a + t\log_3 b$	B1
	$N = 3^{2t+4} = 3^{2t} \times 3^4$ **or** $\log_3 a = 4 \Rightarrow a = \ldots$ **or** $\log_3 b = 2 \Rightarrow b = \ldots$	M1
	$N = 81 \times 9^t$	A1cso **(3)** **(6 marks)**

Question Number	Scheme	Marks
3(a)	$\log_{10} D = 1.04 + 0.38t \Rightarrow D = 10^{1.04+0.38t}$ **or** $a = 10^{1.04}$ or $b = 10^{0.38}$	M1
	$a =$ awrt 10.96 **or** $b =$ awrt 2.399	A1
	$D = 10.96 \times 2.399^t$	A1
		(3)
(b)	$45000 = "10.96" \times "2.399"^T \Rightarrow T = \ldots$ or $\log_{10} 45000 = 1.04 + 0.38T \Rightarrow T = \ldots$	M1
	awrt 9.51	A1
		(2)
(c)	$D = "10.96" \times "2.399"^{12} \Rightarrow D = \ldots$ or $\log_{10} D = 1.04 + 0.38 \times 12 \Rightarrow D = \ldots$ or $350000 = "10.96" \times "2.399"^t \Rightarrow t = \ldots$ or $\log_{10} 350000 = 1.04 + 0.38 \times t \Rightarrow t = \ldots$	M1
	$D =$ awrt (£)400 000 $\Rightarrow$ yes or $t =$ awrt 11.9 $\Rightarrow$ yes	A1
		(2)
		(7 marks)

Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70°}{1+\tan 2x \tan 70°} = -\dfrac{3}{8} \Rightarrow \tan(2x-70°) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right)+70°}{2}$	dM1
	awrt $24.7°$, awrt $114.7°$	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
1(a)	$\mathrm{gf}(1) = \mathrm{g}\left(\dfrac{2(1)}{3\times1+1}\right) = 4 - \left(\dfrac{2(1)}{3\times1+1}\right)^2$	M1
	$= \dfrac{15}{4}$ oe	A1
		(2)
(b)	$\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) < \dfrac{2}{3}$	B1
	$0 \leqslant \mathrm{f}(x) < \dfrac{2}{3}$	B1
		(2)
(c)	$y = \dfrac{2x}{3x+1} \Rightarrow 3xy + y = 2x \Rightarrow 3xy - 2x = -y \Rightarrow x(3y-2) = -y$ or $x = \dfrac{2y}{3y+1} \Rightarrow 3xy + x = 2y \Rightarrow 3xy - 2y = -x \Rightarrow y(3x-2) = -x$	M1
	$\Rightarrow \mathrm{f}^{-1}(x) = \dfrac{x}{2-3x}$	A1
		(2)
(d)	$\mathrm{f}^{-1}(x) = \mathrm{f}(x)$ **or** $\mathrm{f}^{-1}(x) = x$ **or** $\mathrm{f}(x) = x$ $"\dfrac{x}{2-3x}" = \dfrac{2x}{3x+1}$ **or** $"\dfrac{x}{2-3x}" = x$ **or** $\dfrac{2x}{3x+1} = x$ $\Rightarrow x(3x+1) = 2x(2-3x)$ **or** $x = x(2-3x)$ **or** $2x = x(3x+1)$ $\Rightarrow x = \ldots$	M1
	$x = 0, \dfrac{1}{3}$	A1
		(2)
		Total 8

Question Number	Scheme	Marks
7(a)	$y=\dfrac{16}{9(3x-k)}=\dfrac{16}{27x-9k}\Rightarrow\dfrac{dy}{dx}=-\dfrac{432}{(27x-9k)^2}$	M1A0
(b)	$-\dfrac{432}{(27x-9k)^2}=-12\Rightarrow 12(27x-9k)^2=432$	M1
	$12(27x-9k)^2=432\Rightarrow(27x-9k)^2=36\Rightarrow 27-9k=\pm6\Rightarrow k=...$	**d**M1
	$k=\dfrac{7}{3},\ \dfrac{11}{3}$	A1
(c)	$y=\dfrac{16}{27-21}$	M1 (B1 on ePEN)
	$y-\text{"}\dfrac{8}{3}\text{"}=\dfrac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
(d)	$\displaystyle\int\frac{16}{27x-9k}\,dx=\frac{16}{27}[\ln(27x-9k)]$	M1
	$=\dfrac{16}{27}\left[\ln(27x-21)\right]$	A1ft
	$\dfrac{16}{27}\left[\ln(27x-21)\right]_1^3=\dfrac{16}{27}\left(\ln(27(3)-21)-\ln(27-21)\right)$	**d**M1
	$=\dfrac{16}{27}\ln(10)$	A1

Question Number	Scheme	Marks
3(a)	$m=\dfrac{2.25-2}{5}$ $\log_{10}V=0.05t+2$	M1 A1
		(2)
(b)	$\log_{10}V=0.05t+2\Rightarrow V=10^{0.05t+2}$ $a=100$ or $b=1.12$ $V=100\times(1.12)^t$	M1 A1 A1
		(3)
(c)	$\dfrac{dV}{dt}=100\times\ln1.12\times(1.12)^t\quad\left(=11.33(1.12)^t\right)$ $100\times\ln1.12\times(1.12)^T=50\Rightarrow(1.12)^T=\dfrac{50}{100\ln1.12}$ $(T=)\ \log_{1.12}\left(\dfrac{50}{100\ln1.12}\right)=...$ $(T=)13$	B1ft M1 dM1 A1
		(4)
		(9 marks)

(c)	$g^{-1}(x) = e^{\frac{x-3}{2}}$	M1A1
	$x \ldots 3$	B1
		(3)
(d)	$g\left(\frac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\frac{2a}{3a-5}\right) = 5$	B1
	$\ln\left(\frac{2a}{3a-5}\right) = 1 \Rightarrow \frac{2a}{3a-5} = e$	M1
	$\frac{2a}{3a-5} = e \Rightarrow a = \ldots$	dM1
	$a = \frac{5e}{3e-2}$	A1
		(4)

Question Number	Scheme	Marks
8(a)	$25 = a + -(5 \times -2 + b)\ (\Rightarrow 25 = a + 10 - b) \Rightarrow a = 15 + b$ *	M1A1*
		(2)
(b)	$9 = a + 10 + b \Rightarrow a = \ldots$ or $b = \ldots.$ $a = 7,\ b = -8$	M1 A1A1
		(3)
(c)	$\left(\frac{8}{5}, 7\right)$	B1ftB1
		(2)
(d)	$15 - 5x = -2x^3 + 5x^2 + 4x - 3 \Rightarrow 2x^3 - 5x^2 - 9x + 18 = 0$ $2x^3 - 5x^2 - 9x + 18 = (x+2)(2x^2 - 9x + 9)$ $2x^2 - 9x + 9 = 0 \Rightarrow x = \frac{3}{2}$ (ignore $x = 3$) $\left(\frac{3}{2}, \frac{15}{2}\right)$	M1 dM1A1 ddM1 M1A1
		(6)
		(13 marks)

1. (a)	$g(3) = -265, \quad g(4) = 3104$ States change of sign, continuous and hence root in $[3,4]$	M1 A1 (2)
(b)	$x_2 = \sqrt[6]{1000 - 2\times 3} \quad = 3.1591$ $(\alpha =)3.1589$	M1 A1 A1 (3)
		(5 marks)

Question Number	Scheme	Marks
9(a)	$y=\sqrt{3+4e^{x^2}}=\left(3+4e^{x^2}\right)^{\frac{1}{2}}\Rightarrow\frac{dy}{dx}=\frac{1}{2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}\times 8xe^{x^2}$	M1
	$=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\frac{\left(3+4e^{x^2}\right)}{x}=4xe^{x^2}$	dM1
	$4x^2e^{x^2}-4e^{x^2}-3=0$ *	A1*
		(3)
(c)	$f(x)=4x^2e^{x^2}-4e^{x^2}-3\Rightarrow f(1)=-3$ AND $f(2)=652\cdot\ldots$	M1
	Change of sign and $f(x)$ is continuous hence root in $(1, 2)$	A1
		(2)
(d)	$4x^2e^{x^2}-4e^{x^2}-3=0\Rightarrow x^2=\frac{4e^{x^2}+3}{4e^{x^2}}=\frac{4+3e^{-x^2}}{4}\Rightarrow x=\frac{1}{2}\sqrt{4+3e^{-x^2}}$ *	B1*
		(1)
(e)(i)	$x_1=1\Rightarrow x_2=\frac{1}{2}\sqrt{4+3e^{-1}}$	M1
	$x_3=1.0997$	A1
(ii)	$\alpha=1.1051$	A1
		(3)
		Total 11

1	$3\tan^2\theta+7\sec\theta-3=0\Rightarrow 3\left(\sec^2\theta-1\right)+7\sec\theta-3=0$	M1
	$3\sec^2\theta+7\sec\theta-6=0$	A1
	$(3\sec\theta-2)(\sec\theta+3)=0\Rightarrow\sec\theta=\ldots\Rightarrow\cos\theta=\ldots$	dM1
	$\theta=109.5°, 250.5°$	A1, A1
		(5)

Question Number	Scheme	Marks
6(i)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{4x}{2x^2+5}$	M1A1
		(2)
(ii)	$\int \frac{21x}{3x^2+k}\,\mathrm{d}x=\frac{7}{2}\ln(3x^2+k)\ \ (+c)$	M1A1
	$\left[\frac{7}{2}\ln(3x^2+k)\right]_1^k<7\ln 8 \Rightarrow \frac{7}{2}\ln\left(\frac{3k^2+k}{3+k}\right)<7\ln 8 \Rightarrow \frac{3k^2+k}{3+k}<64$	M1
	$\frac{3k^2+k}{3+k}(<)64 \Rightarrow 3k^2-63k-192\ \ (<0) \Rightarrow k=\ldots$	dM1
	$k=23$	A1
		(5)
		(7 marks)

10 (a)	$x=\frac{2y^2+6}{3y-3} \Rightarrow \left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)\frac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{6y^2-12y-18}{9(y-1)^2}=\frac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\frac{\mathrm{d}x}{\mathrm{d}y}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0 \Rightarrow 2(y-3)(y+1)=0 \Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\frac{2y^2+6}{3y-3} \Rightarrow x=..$	dM1
	Achieves $x=-\frac{4}{3}$ and $x=4$	A1cso
		(4)
		8 marks

2	$\mathrm{g}(x)=\frac{2x^2-5x+8}{x-2}$	
(a)	Sets $2x^2-5x+8=(Ax+B)(x-2)+C$ with an attempt at one constant	M1
	Attempts all 3 constants. E.g. Sets $x=2 \Rightarrow C=\ldots,-2B+C=8 \Rightarrow B=\ldots$ and compares x^2 terms gives $A=\ldots$	dM1
	$2x-1+\frac{6}{x-2}$	A1
		(3)

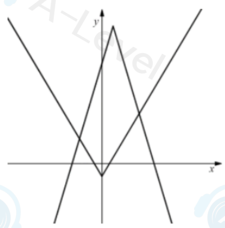
(b)	$\int 2x-1+\frac{6}{x-2}\,dx = x^2 - x + 6\ln(x-2)$	M1 A1ft
	$\int_4^8 2x-1+\frac{6}{x-2}\,dx = \left[x^2 - x + 6\ln(x-2)\right]_4^8 = 56+6\ln 6-12-6\ln 2$	
	$= 44 + 6\ln 3$	dM1 A1
		(4)
		(7 marks)

4.(a)	$$\begin{array}{r} 3x-5 \\ x^2+0x+1\overline{\big)3x^3-5x^2+7x-5} \\ \underline{3x^3+0x^2+3x} \\ -5x^2+4x \\ \underline{-5x^2+0x-5} \\ 4x \end{array}$$	M1
	$\underline{A=3}$, $B=-5, C=4$	<u>B1</u>, A1
	$D=0$ *	A1*
		(4)
(b)	$\int 3x-5+\frac{4x}{x^2+1}\,dx = \frac{3x^2}{2} - 5x + 2\ln(x^2+1) + c$	M1, A1ft
	Area $= \left(\frac{3\times3^2}{2} - 5\times3 + 2\ln(3^2+1)\right) - \left(\frac{3\times2^2}{2} - 5\times2 + 2\ln(2^2+1)\right)$	
	$= \frac{5}{2} + \ln 4$	dM1, A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
1(a)	$(f(1)=)1-5+e=-1.281...<0$ $(f(2)=)2^2-5\times2+e^2=1.389...>0$	M1
	As there is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> over the interval $[1, 2]$ then <u>there is a root</u> *	A1*
		(2)
(b)(i)	$x_2 = \sqrt{5\times1-e^1} = \text{awrt}\,1.5105$	M1A1
(ii)	$\alpha = \text{awrt } 1.7340$	A1
		(3)
		(5 marks)

Question Number	Scheme	Marks
1 (a)	$(5,10)$	B1 B1
		(2)
(b)	Attempts to solve e.g. $-2(x-5)+10 ... 6x \Rightarrow x ... "\frac{5}{2}"$	M1
	$x<\frac{5}{2}$ o.e.	A1
		(2)
(c)	$(7,30)$	B1ft, B1ft
		(2)
		(6 marks)

Question Number	Scheme	Marks
8(a)(i)	$\left(\frac{b}{2},\ a\right)$	B1B1
(ii)	$(0,\ a-b)$	B1
(iii)	$\left(\frac{b-a}{2},\ 0\right)$ and $\left(\frac{a+b}{2},\ 0\right)$	B1B1
		(5)

(b)		B1B1
		(2)
(c)	$-x-1=2x+a-b,\ x=-3 \Rightarrow 2=-6+a-b$ or $x-1=a+b-2x,\ x=5 \Rightarrow 5-1=a+b-10$	M1
	$-x-1=2x+a-b,\ x=-3 \Rightarrow 2=-6+a-b$ and $x-1=a+b-2x,\ x=5 \Rightarrow 5-1=a+b-10$	dM1 (A1 on ePEN)
	$\begin{matrix} a-b=8 \\ a+b=14 \end{matrix} \Rightarrow a= ...$ or $b= ...$	**dd**M1
	$a=11,\ b=3$	A1
		(4)
		(11 marks)

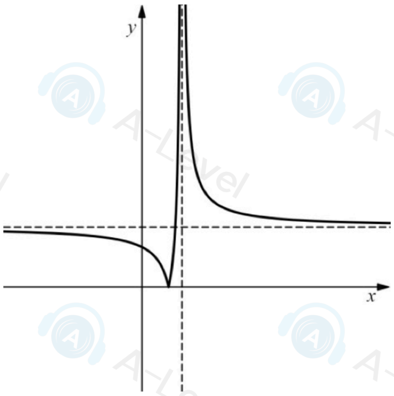
7.(a)	States or implies that $A = 2\,500$	B1
	$10000 = 2500e^{k\times 8} \Rightarrow 8k = \ln 4 \Rightarrow k = ...$	M1
	$\Rightarrow k = \frac{1}{8}\ln 4$ or awrt 0.1733	A1
		(3)
(b)	$\frac{dN}{dt} = 60000 \times -0.6e^{-0.6\times 5} = -1792$ So decrease is 1790	M1, A1
		(2)
(c)	$60000e^{-0.6t} = 2500e^{0.1733t}$	M1
	$24 = e^{0.1733t+0.6t} \Rightarrow 0.1733t + 0.6t = \ln 24 \Rightarrow t = ...$	dM1
	$T = 4.11$	A1
		(3)
		8 marks

7 (a)	$\sin 4\theta \equiv 2\sin 2\theta \cos 2\theta$	M1
	$\equiv 2(2\sin\theta\cos\theta)(1-2\sin^2\theta)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta(4-8\sin^2\theta)$	A1
		(3)
(b)	$\sec x \sin 4x = 5\sin^3 x \cot x$	
	$\frac{1}{\cos x} \times \cos x \sin x(4-8\sin^2 x) = 5\sin^3 x \cot x$	B1
	$\div \sin x \Rightarrow 4-8\sin^2 x = 5\sin^2 x \cot x$	
	$\div \cos^2 x \Rightarrow 4\sec^2 x - 8\tan^2 x = 5\tan^2 x \cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$ *	A1*
		(3)
(c)	Uses $\sec^2 x = 1+\tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = ...$	dM1
	$\Rightarrow x =$ awrt 0.506, 2.08	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
2(a)	$(\mathrm{f}(2)=)2^4-5(2)^2+4(2)-7=-3<0$ **and** $(\mathrm{f}(3)=)3^4-5(3)^2+4(3)-7=41>0$	M1
	There is a <u>change of sign</u> and $\mathrm{f}(x)$ is <u>continuous</u> so <u>there is a root</u> (in the interval) *	A1*
		(2)
(b)	$x^3=\dfrac{5x^2-4x+7}{x}\Rightarrow x=\sqrt[3]{\dfrac{5x^2-4x+7}{x}}$ *	B1*
		(1)
(c)(i)	$x_2=\sqrt[3]{\dfrac{5(2)^2-4(2)+7}{2}}$	M1
	$=$ awrt 2.1179	A1
(ii)	$\alpha=$ awrt 2.1565	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(x)\geqslant\ln 3$	B1
		(1)
(b)	$\mathrm{g}^{-1}(x)=\dfrac{3-2x}{x-5}$ or $-2-\dfrac{7}{x-5}$ oe $x<5$	M1A1 B1
		(3)
(c)	$\mathrm{g}(0)=\dfrac{3}{2}\Rightarrow \mathrm{f}\left(\dfrac{3}{2}\right)=\ln\left(\dfrac{21}{4}\right)$	M1A1
		(2)
(d)	$\dfrac{3+5\mathrm{e}^{2a}}{\mathrm{e}^{2a}+2}=4\Rightarrow 5\mathrm{e}^{2a}+3=4\mathrm{e}^{2a}+8\Rightarrow \mathrm{e}^{2a}=\ldots$ $\mathrm{e}^{2a}=5\Rightarrow 2a=\ln 5\Rightarrow a=\ldots$ $a=\dfrac{1}{2}\ln 5$	M1 dM1 A1
		(3)
		(9 marks)

(a)

8. (a)	$g(2a)=\dfrac{3\times 2a-2a}{2a-a}=4,\quad f(4)=\ln(2\times 4-3)=\ln 5$	M1, A1
		(2)
(b)	$y=\ln(2x-3)\Rightarrow e^{y}=2x-3\Rightarrow x=\dfrac{e^{y}+3}{2}$	M1
	$f^{-1}(x)=\dfrac{e^{x}+3}{2}\qquad x>0$	A1, B1
		(3)
(c)	 Shape of curve to right of $x=a$	B1
	Shape of curve to left of $x=a$	B1
	y =3 marked at correct place on curve	B1
		(3)
(d) (i)	$\dfrac{3\times 8-2a}{8-a}=10\Rightarrow a=\ldots$	M1
	$\Rightarrow a=7$	A1
(ii)	$\dfrac{3x-2\times 7}{x-7}=-10\Rightarrow x=\ldots$	dM1
	$\Rightarrow x=\dfrac{84}{13}$	A1
		(4)
		(12 marks)

Question Number	Scheme	Marks
4(a)	$R=\sqrt{12}$ or $2\sqrt{3}$	B1
	$\alpha=\tan^{-1}\left(\frac{3}{\sqrt{3}}\right)=\ldots$	M1
	$(\mathrm{f}(x)=)\sqrt{12}\sin\left(2x-\frac{\pi}{3}\right)$	A1
		(3)
(b)(i)	Minimum value $=\dfrac{18}{\sqrt{12}+4\sqrt{3}}\left(=\sqrt{3}\right)$	B1
(ii)	$\sin\left(6x-\frac{\pi}{3}\right)=1\Rightarrow x=\frac{5}{36}\pi$	M1A1
		(3)
		(6 marks)

8	$h=1.5x-0.5x\mathrm{e}^{0.02x}$	
(a)	$0=1.5d-0.5d\,\mathrm{e}^{0.02d}\Rightarrow\mathrm{e}^{0.02d}=3$	M1
	$\Rightarrow 0.02d=\ln 3\Rightarrow d=54.93$	dM1 A1
		(3)
(b)	$\left(\frac{\mathrm{d}h}{\mathrm{d}x}=\right)1.5-\left(0.5\mathrm{e}^{0.02x}+0.5x\times0.02\mathrm{e}^{0.02x}\right)$	M1A1
	Sets $0=1.5-0.5\mathrm{e}^{0.02x}-0.5x\times0.02\mathrm{e}^{0.02x}\Rightarrow\mathrm{e}^{0.02x}(0.5+0.5x\times0.02)=1.5$	
	$\Rightarrow\mathrm{e}^{0.02x}=\dfrac{1.5}{(0.5+0.5x\times0.02)}$	dM1
	$\Rightarrow\mathrm{e}^{0.02x}=\dfrac{1.5}{(0.5+0.5x\times0.02)}=\dfrac{150}{50+x}\Rightarrow x=50\ln\left(\dfrac{150}{50+x}\right)$ *	A1*
		(4)
(c)	(i) awrt 31.43	M1 A1
	(ii) awrt 30.88 metres **(including units)**	A1
		(3)
		(10 marks)

5(i)	$\cos 6x=1-2\sin^2 3x\Rightarrow\sin^2 3x=\frac{1}{2}-\frac{1}{2}\cos 6x$ $\int\sin^2 3x\,\mathrm{d}x=\int\left(\frac{1}{2}-\frac{1}{2}\cos 6x\right)\mathrm{d}x$	M1
	$=\frac{1}{2}x-\frac{1}{12}\sin 6x(+c)$	A1
		(2)
(ii)	$\int x\left(x^2+4\right)^{\frac{3}{2}}\mathrm{d}x=\frac{1}{5}\left(x^2+4\right)^{\frac{5}{2}}(+c)$	M1A1
		(2)
		Total 4

8. (a)	$f'(x)=6(2x+1)^2 e^{-4x}-4(2x+1)^3 e^{-4x}$	M1 A1
	$=2(2x+1)^2 e^{-4x}\{3-2(2x+1)\}$	dM1
	$=2(2x+1)^2(1-4x)e^{-4x}$	A1
		(4)
(b)	Sets $f'(x)=0 \Rightarrow x=-\frac{1}{2},\frac{1}{4}$	B1
	Either $f\left("-\frac{1}{2}"\right)=...$ or $f\left("\frac{1}{4}"\right)=...$	M1
	Both $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$	A1
		(3)
(c)	$\left(\frac{9}{4},\frac{27}{e}\right)$	B1ft B1ft
		(2)
		9 marks

Question Number	Scheme	Marks
6(a)	$\theta=75,\ t=0 \Rightarrow 75=21+A \Rightarrow A=...$	M1
	$A=54$	A1
		(2)
(b)	$\theta=21+54e^{-kt} \Rightarrow 25=21+54e^{-5k}$	M1
	$54e^{-5k}=4 \Rightarrow e^{-5k}=\frac{2}{27} \Rightarrow -5k=\ln\frac{2}{27} \Rightarrow k=...$	M1
	$k=-\frac{1}{5}\ln\frac{2}{27}=0.521$	A1
		(3)
(c)	$\theta=21+54e^{-0.521t} \Rightarrow \frac{d\theta}{dt}=-28.1...e^{-0.521t}$	M1
	$-28.1...e^{-0.521T}=-9 \Rightarrow e^{-0.521T}=\frac{9}{28.1...} \Rightarrow -0.521T=\ln\left(\frac{9}{28.1...}\right)$ $\Rightarrow T=\ln\left(\frac{9}{28.1...}\right)\div -0.521$	dM1
	$=2.19$	A1
		(3)

Question Number	Scheme	Marks
1(a)	$(-2,-3)$	B1
		(1)
(b)	$(-3,-9)$	B1B1
		(2)
(c)	$(-4,3)$	B1
		(1)
		(4 marks)

10(a)	$\frac{1}{4}=\sin^2 4y \Rightarrow y=\frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y$	M1A1
		(2)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y \to \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x=\frac{1}{2}$	B1ft
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}$	B1ft
		(2)
		(9 marks)

5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+k)\times\frac{2x}{x^2+k}-2x\ln(x^2+k)}{(x^2+k)^2}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-2x\ln(x^2+k)}{(x^2+k)^2}=\frac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$	M1A1
		(3)
(b)	$x=0$	B1
	$"1"-\ln(x^2+k)=0 \Rightarrow x^2=\mathrm{e}^{"1"}\pm k$	M1
	$x=\pm\sqrt{\mathrm{e}-k}$	A1ft
		(3)
(c)	Upper limit is e or $k<\mathrm{e}$	B1ft
		(1)
		(7 marks)

4(a)	$A=93$	B1
		(1)
(b)	$100=125-"93"\mathrm{e}^{-0.109T} \Rightarrow A\mathrm{e}^{-0.109T}=\ldots$	M1
	$A\mathrm{e}^{kT}=B \Rightarrow T=\frac{\ln\left(\frac{B}{A}\right)}{k}$	dM1
	$T=12.05$	A1
		(3)
(c)	$\frac{\mathrm{d}N}{\mathrm{d}t}=0.109\times"93"\mathrm{e}^{-0.109\times 7}$	M1
	4 730 (total sales per month)	A1
		(2)
(d)	The limit is 125 000 / the model has a limit below 150 000	B1
		(1)
		(7 marks)

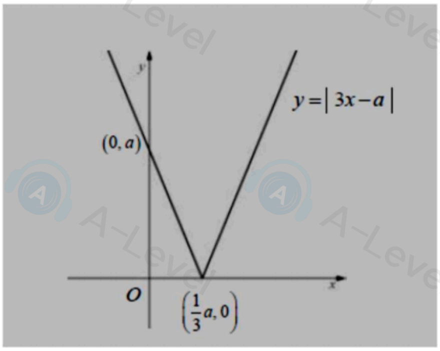
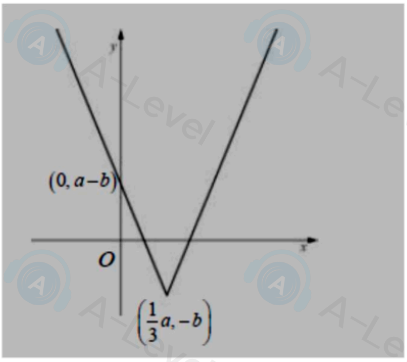
Question Number	Scheme	Marks
5(a)	$\left(\dfrac{4000e^0}{19+e^0}\right)=200$	B1
		(1)
(b)	$\dfrac{dN}{dt}=\dfrac{\left(19+e^{0.2t}\right)\times 400e^{0.1t}-4000e^{0.1t}\times 0.2e^{0.2t}}{\left(19+e^{0.2t}\right)^2}$	M1A1
		(2)
(c)	$\left(19+e^{0.2t}\right)\times 400e^{0.1t}-4000e^{0.1t}\times 0.2e^{0.2t}=0\Rightarrow e^{0.2T}=19$	M1A1
		(2)
(d)	$e^{0.2T}=19\Rightarrow T=\dfrac{\ln 19}{0.2}\ (=14.7...)$	M1
	$N=\dfrac{4000e^{0.1"14.7"}}{19+e^{0.2"14.7"}}=458.8...\Rightarrow 459$ squirrels	dM1A1
		(3)
		(8 marks)

5 (a)	$f^{-1}(22)\Rightarrow 2+5\ln x=22\Rightarrow \ln x=4\Rightarrow (x=)\ e^4$	M1 A1
		(2)
(b)	$g(x)=\dfrac{6x-2}{2x+1}\Rightarrow g'(x)=\dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$	M1 A1
	States $(g'(x)=)\ \dfrac{10}{(2x+1)^2}>0$ hence increasing *	A1*
		(3)
(c)	$y=\dfrac{6x-2}{2x+1}\Rightarrow 2xy+y=6x-2\Rightarrow 2xy-6x=-y-2$	M1
	$\Rightarrow x=\dfrac{-y-2}{2y-6}$ So $g^{-1}(x)=\dfrac{-x-2}{2x-6}$ o.e.	A1
	Domain $0<x<3$	B1
		(3)
(d)	Range fg is $fg<2+5\ln 3$	M1, A1
		(2)
		(10 marks)

8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \dfrac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1-\tan^2 x}\tan x} \equiv \dfrac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2\tan^2 x}$ $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ *	A1*
		(3)

(b)	$\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 2\sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2\tan^2 3\theta - \tan 3\theta - 6 = 0$ or e.g. $\Rightarrow 2\tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2\tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\dfrac{3}{2},\ 2$ $\tan 3\theta = -\dfrac{3}{2} \Rightarrow 3\theta = ... \Rightarrow \theta = ...$ or $\tan 3\theta = 2 \Rightarrow 3\theta = ... \Rightarrow \theta = ...$	dM1
	$\theta = 0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

Question Number	Scheme	Marks
6(a)	$\left(\dfrac{3\pi}{2},\ 0\right)$	B1
		(1)
(b)	$f'(x) = (6\cos^2 x)e^{3\sin x} - (2\sin x)e^{3\sin x}$	M1A1
	$f'(x) = (6\cos^2 x)e^{3\sin x} - (2\sin x)e^{3\sin x}$ $\Rightarrow (6(1-\sin^2 x) - 2\sin x)(=0)$	**d**M1
	$\Rightarrow 3\sin^2 x + \sin x - 3 = 0$ oe	A1
		(4)
(c)	$\sin x = \dfrac{-1+\sqrt{37}}{6} \Rightarrow x = ...$	M1
	$x = 2.131$	A1
		(2)
		(7 marks)

7 (a)(i)	 Shape Vertex $\left(\frac{1}{3}a,0\right)$ y intercept $(0,a)$	B1 B1 B1
(a)(ii)	 Shape Vertex $\left(\frac{1}{3}a,-b\right)$ y intercept $(0,a-b)$	B1ft B1ft B1ft **(6)**
(b)	Attempts to solve $-3x+a-b=5x\Rightarrow x=\ldots$ $\Rightarrow x=\dfrac{a-b}{8}$ ONLY	M1 A1 **(2)** **(8 marks)**

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(x)\leqslant 9$	B1
		(1)
(b)	$\mathrm{fg}(1.5)=\mathrm{f}\left(\dfrac{3}{2\times1.5+1}\right)=9-\left(\dfrac{3}{2\times1.5+1}\right)^2$	M1
	$=\dfrac{135}{16}$	A1
		(2)
(c)	$\mathrm{g}(x)=\dfrac{3}{2x+1}\Rightarrow \mathrm{g}^{-1}(x)=\dfrac{3-x}{2x}$	M1 A1
	$0<x\leqslant 3$	B1
		(3)
		Total 6

5.(a)	$\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$	M1, A1
	$\left.\frac{dy}{dx}\right\vert_{x=\frac{\pi}{3}} = -10\sin\left(\frac{2\pi}{3}\right) - 24\cos\left(\frac{2\pi}{3}\right) = -5\sqrt{3} + 12$	A1
		(3)
(b)	$R = 13$	B1
	$\tan\alpha = \frac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	M1A1
		(3)
(c)	Sets $-26\sin(2x + 1.176) = 6$	M1
	$\sin(2x + "1.176") = -\frac{3}{13}$	A1 ft
	$x = \dfrac{\arcsin\left(-\frac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	dM1, A1
		(4)
		(10 marks)

6 (a)	$y = \frac{4x+3}{x-2} \Rightarrow yx - 2y = 4x + 3$ $\Rightarrow yx - 4x = 2y + 3$	$y = 4 + \frac{11}{x-2} \Rightarrow y - 4 = \frac{11}{x-2}$ $x - 2 = \frac{11}{y-4}$	M1
	$\Rightarrow x = \frac{2y+3}{y-4} \Rightarrow f^{-1}(x) = \frac{2x+3}{x-4}$	$\Rightarrow x = 2 + \frac{11}{y-4} \Rightarrow f^{-1}(x) = 2 + \frac{11}{x-4}$	A1
	$x \neq 4$		B1
			(3)
(b)	$ff(x) = \dfrac{4 \times \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 2}$ or $ff(x) = 4 + \dfrac{11}{4 + \frac{11}{x-2} - 2}$		M1
	$= \dfrac{4 \times (4x+3) + 3(x-2)}{4x + 3 - 2(x-2)} = \dfrac{19x+6}{2x+7}$		dM1 A1
			(3)
(c)	Either $x = 1$ or $y = 38$		M1
	$(1, 38)$		A1
			(2)
			(8 marks)

Question Number	Scheme	Marks
3. (a)	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{8y(2y+1)-2(4y^2-3)}{(2y+1)^2}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2y+1)^2}{8y^2+8y+6}$	dM1, A1
		(4)
(b)	Sets $\frac{(2y+1)^2}{8y^2+8y+6}=\frac{1}{3}\Rightarrow 4y^2+4y-3=0$	M1, A1
	$\Rightarrow(2y-1)(2y+3)=0\Rightarrow y=\frac{1}{2},x=\ldots$	dM1
	$P=\left(-1,\ \frac{1}{2}\right)$ only	A1
		(4)
		(8 marks)