

1	$3 \tan^2 \theta + 7 \sec \theta - 3 = 0 \Rightarrow 3(\sec^2 \theta - 1) + 7 \sec \theta - 3 = 0$	M1
	$3 \sec^2 \theta + 7 \sec \theta - 6 = 0$	A1
	$(3 \sec \theta - 2)(\sec \theta + 3) = 0 \Rightarrow \sec \theta = \dots \Rightarrow \cos \theta = \dots$	dM1
	$\theta = 109.5^\circ, 250.5^\circ$	A1, A1
		(5)

7 (a)	$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$	M1
	$\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$	dM1
	$\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$	
	$\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$	A1
		(3)
(b)	$\sec x \sin 4x = 5 \sin^3 x \cot x$	
	$\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$	B1
	$\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$	
	$\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$	M1
	$\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$	A1*
		(3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$	M1
	$\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$	dM1
	$\Rightarrow x = \text{awrt } 0.506, 2.08$	A1
		(4)
		(10 marks)

4(a)	$f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$	
	States or uses $\sin 2x = 2 \sin x \cos x$ or $\cos 2x = \pm 2 \cos^2 x \pm 1$	M1
	Uses $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \pm 2 \cos^2 x \pm 1$ in $f(x)$	dM1
	$(f(x) =) 8 \sin x \cos x + 4 \cos^2 x - 3 = 4 \sin 2x + 2 \cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$	B1ft
	$\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots$ ("awrt 0.464")	M1
	$(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$	A1
		(3)
(c)	(i) Maximum value = " $2\sqrt{5} - 1$ "	B1 ft
	(ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$	M1
	$(x =) \text{awrt } 3.69 \text{ (or } (x =) \text{awrt } 3.70)$	A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\dots} \text{ or } \frac{\dots}{\left(\frac{1}{2} \sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2} \sin 2x\right)^2}$	A1
	$\equiv 4 \frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4 \cot 2x \operatorname{cosec} 2x^*$	A1*
	(4)	
(b)	$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - \tan^2 \theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - 3 \tan^2 \theta = 0$	M1
	$\cot^2 \theta - 3 \tan^2 \theta = 0 \Rightarrow \frac{1}{\tan^2 \theta} - 3 \tan^2 \theta = 0 \Rightarrow \tan^4 \theta = \frac{1}{3}$	A1
	$\tan^4 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \sqrt[4]{\frac{1}{3}} = \pm 0.7598.. \Rightarrow \theta = \dots$	M1
	$\theta = \text{awrt } 0.65, -0.65$	A1A1
	(5)	
	Total 9	

8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x+x)}{\cos(2x+x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1
	$\frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \equiv \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$ $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}^*$	A1*
	(3)	

(b)	$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2 \sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2 \tan^2 3\theta - \tan 3\theta - 6 = 0 \quad \text{or e.g.} \Rightarrow 2 \tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2 \tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots \quad \text{or} \quad \tan 3\theta = 2 \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
		Total 8

5 (i)	States $x = 2$	B1
	$\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$	M1
(ii)	$x = \frac{5\pi}{6}$	A1
	Attempts to use $\cos 2\theta = 1 - 2\sin^2 \theta$	M1
	$6\sin^2 \theta + 10\sin \theta - 3 = 0$	A1
	$\sin \theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$	M1
	$\theta = 15.0^\circ, 165^\circ$	A1
		(4)
		(7 marks)