

10(a)	$\frac{1}{4} = \sin^2 4y \Rightarrow y = \frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{dx}{dy} = \underline{\underline{8 \sin 4y \cos 4y}}$	<u>M1A1</u>
		(2)
(c)	$\frac{dx}{dy} = 8 \sin 4y \cos 4y \rightarrow \frac{dy}{dx} = \frac{1}{8 \sin 4y \cos 4y}$	M1
	$\frac{dy}{dx} = \frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x = \frac{1}{2}$	B1ft
(ii)	$\frac{dy}{dx} = \frac{1}{4}$	B1ft
		(2)
		(9 marks)

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$	M1 A1
	$\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$	dM1, A1
		(4)
(b)	Sets $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 4y^2 + 4y - 3 = 0$	M1, A1
	$\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \dots$	dM1
	$P = \left(-1, \frac{1}{2}\right)$ only	A1
		(4)
		(8 marks)

10.(a)	$x = 3 \cos 2y \Rightarrow \left(\frac{dx}{dy} \right) = -6 \sin 2y$	M1 A1 (2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$ $\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	M1 dM1 A1 (3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$ $x^2 = 5 \Rightarrow a = \sqrt{5}$ $\sqrt{5} = 3 \cos 2y \Rightarrow b = \frac{1}{2} \arccos\left(\frac{\sqrt{5}}{3}\right)$	M1 A1 dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6 \sin 2y} = -\frac{1}{4}$ $\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2} \arcsin\left(\frac{2}{3}\right)$ $x = 3 \cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	M1A1 dM1 A1 (4) (9 marks)

Question Number	Scheme	Marks
6 (a)	$y = (4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} \right) = 2(4x-7)^{-\frac{1}{2}}$ (see notes) At $(8,5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} = \frac{2}{5}$ Equation for l is $y-5 = -\frac{5}{2}(x-8)$ $2y-10 = -5x+40 \Rightarrow 5x+2y-50=0$ *	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{6} \right]$ (see notes) Complete area = $\int_{\frac{7}{4}}^8 (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{6} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

Question Number	Scheme	Marks
6(i)	$\left(\frac{dy}{dx}\right) = \frac{4x}{2x^2+5}$	M1A1
		(2)
(ii)	$\int \frac{21x}{3x^2+k} dx = \frac{7}{2} \ln(3x^2+k) (+c)$ $\left[\frac{7}{2} \ln(3x^2+k)\right]_1^k < 7 \ln 8 \Rightarrow \frac{7}{2} \ln\left(\frac{3k^2+k}{3+k}\right) < 7 \ln 8 \Rightarrow \frac{3k^2+k}{3+k} < 64$ $\frac{3k^2+k}{3+k} (<) 64 \Rightarrow 3k^2 - 63k - 192 (< 0) \Rightarrow k = \dots$ $k = 23$	M1A1 M1 dM1 A1
		(5)
		(7 marks)

Question Number	Scheme	Marks
7(a)	$y = \frac{16}{9(3x-k)} = \frac{16}{27x-9k} \Rightarrow \frac{dy}{dx} = -\frac{432}{(27x-9k)^2}$	M1A0
(b)	$-\frac{432}{(27x-9k)^2} = -12 \Rightarrow 12(27x-9k)^2 = 432$	M1
	$12(27x-9k)^2 = 432 \Rightarrow (27x-9k)^2 = 36 \Rightarrow 27-9k = \pm 6 \Rightarrow k = \dots$	dM1
	$k = \frac{7}{3}, \frac{11}{3}$	A1
(c)	$y = \frac{16}{27-21}$	M1 (B1 on ePEN)
	$y - \frac{8}{3} = \frac{1}{12}(x-1)$	dM1
	$12y - x - 31 = 0$	A1
(d)	$\int \frac{16}{27x-9k} dx = \frac{16}{27} [\ln(27x-9k)]$	M1
	$= \frac{16}{27} [\ln(27x-21)]$	A1ft
	$\frac{16}{27} [\ln(27x-21)]_1^3 = \frac{16}{27} (\ln(27(3)-21) - \ln(27-21))$	dM1
	$= \frac{16}{27} \ln(10)$	A1

6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ $= 2(2x-3)^2 e^{4x-2} \{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	M1 A1 dM1 A1 (4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$ $-27e, g(x), 0g g \text{ ;}$	M1, A1 A1 (4) (8 marks)

9(a)	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x}{\sin x} + \frac{2 \sin x \cos x}{\cos x} \quad (\text{One Correct identity})$ $= \frac{1 - 2 \sin^2 x}{\sin x} + \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}}$ $= \frac{1}{\sin x} - \frac{2 \sin^2 x}{\sin x} + 2 \sin x = \frac{1}{\sin x} = \operatorname{cosec} x \quad *$	B1 M1 A1* (3)
(b)	E.g. Equation is $\operatorname{cosec}^2 \theta = 6 \cot \theta - 4 \Rightarrow 1 + \cot^2 \theta = 6 \cot \theta - 4$ E.g. $\cot^2 \theta - 6 \cot \theta + 5 = 0$ E.g. $\tan \theta = \frac{1}{5}, 1$ $\theta = 0.197, \frac{\pi}{4}$	M1 A1 dM1 A1, A1 (5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \cot x \, dx = \left[-\operatorname{cosec} x \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ $= 2 - \sqrt{2}$	M1 A1 (2) 10 marks

8. (a)	$f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ $= 2(2x+1)^2 e^{-4x} \{3 - 2(2x+1)\}$ $= 2(2x+1)^2 (1-4x) e^{-4x}$	M1 A1 dM1 A1 (4)
(b)	Sets $f'(x) = 0 \Rightarrow x = -\frac{1}{2}, \frac{1}{4}$ Either $f\left(-\frac{1}{2}\right) = \dots$ or $f\left(\frac{1}{4}\right) = \dots$ Both $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$	B1 M1 A1 (3)
(c)	$\left(\frac{9}{4}, \frac{27}{e}\right)$	B1ft B1ft (2) 9 marks

9 (a)	$(k =) 4\sin^2\left(\frac{\pi}{3}\right) - 1 = 2$ *	B1*
		(1)
(b)	(i) $x = 4\sin^2 y - 1 \Rightarrow \frac{dx}{dy} = 8\sin y \cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\cos^2 y = \pm 1 \pm \frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx} = \frac{1}{8\sin y \cos y} = \frac{1}{8 \times \sqrt{\frac{x+1}{4}} \times \sqrt{\pm 1 \pm \frac{x+1}{4}}} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x = 2$, gradient of curve $= \frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of	
	$\cancel{y} - \frac{\pi}{3} = -2\sqrt{3}(x - 2) \Rightarrow x = 2 + \frac{\pi}{6\sqrt{3}}$ (= 2.3...)	dM1
	Area $= \frac{1}{2} \times \left(2 + \frac{\pi}{6\sqrt{3}}\right) \times \frac{\pi}{3} = \frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x) = \frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x} = 0 \Rightarrow \ln(4x) = -2$	dM1
	$Q\left(\frac{1}{4e^2}, -\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2 \times y$ co-ordinate of Q	M1
	Range $g(x) \leq \frac{12}{e}$	A1ft
		(2)
		(8 marks)