

Question Number	Scheme	Marks
7	$\left(\frac{dy}{dx}\right) = 2e^{x^2+(3k-2)x} + 2xe^{x^2+(3k-2)x} \times (2x + (3k-2))$ $\Rightarrow 2x^2 + (3k-2)x + 1 = 0 \Rightarrow (3k-2)^2 - 4 \times 2 \times 1$ $9k^2 - 12k - 4 (> 0)$ $9k^2 - 12k - 4 = 0 \Rightarrow k = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 9 \times (-4)}}{2 \times 9} \Rightarrow k = \dots$ $k < \frac{2-2\sqrt{2}}{3} \text{ or } k > \frac{2+2\sqrt{2}}{3}$	M1A1A1 dM1A1 M1A1 (7 marks)

2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)

4(a)	$\frac{49x}{x^2+x-12} + \frac{7x}{x+4} = \frac{49x+7x(x-3)}{(x+4)(x-3)}$ or e.g. $\frac{49x(x+4)}{(x^2+x-12)(x+4)} + \frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)} = \frac{49x(x+4)+7x(x^2+x-12)}{(x+4)(x^2+x-12)}$ $\frac{49x+7x(x-3)}{(x+4)(x-3)} = \frac{49x+7x^2-21x}{(x+4)(x-3)} = \frac{7x^2+28x}{(x+4)(x-3)}$ $= \frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} = \frac{7x}{(x-3)}^*$	M1A1 A1* (3)
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(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$	M1 A1 dM1, A1 (4)
(b)	Sets $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 4y^2 + 4y - 3 = 0$ $\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \dots$ $P = \left(-1, \frac{1}{2}\right)$ only	M1, A1 dM1 A1 (4) (8 marks)

Question Number	Scheme	Marks
9(a)	$y = \sqrt{3 + 4e^{x^2}} = (3 + 4e^{x^2})^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(3 + 4e^{x^2})^{-\frac{1}{2}} \times 8xe^{x^2}$	M1
	$= 4xe^{x^2} (3 + 4e^{x^2})^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{(3 + 4e^{x^2})^{\frac{1}{2}}}{x} = 4xe^{x^2} (3 + 4e^{x^2})^{-\frac{1}{2}}$	M1
	$\frac{(3 + 4e^{x^2})}{x} = 4xe^{x^2}$	dM1
	$4x^2e^{x^2} - 4e^{x^2} - 3 = 0^*$	A1*
		(3)
(c)	$f(x) = 4x^2e^{x^2} - 4e^{x^2} - 3 \Rightarrow f(1) = -3 \text{ AND } f(2) = 652 \dots$	M1
	Change of sign and $f(x)$ is continuous hence root in (1, 2)	A1
		(2)
(d)	$4x^2e^{x^2} - 4e^{x^2} - 3 = 0 \Rightarrow x^2 = \frac{4e^{x^2} + 3}{4e^{x^2}} = \frac{4 + 3e^{-x^2}}{4} \Rightarrow x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}^*$	B1*
		(1)
(e)(i)	$x_1 = 1 \Rightarrow x_2 = \frac{1}{2}\sqrt{4 + 3e^{-1}}$	M1
	$x_3 = 1.0997$	A1
(ii)	$\alpha = 1.1051$	A1
		(3)
		Total 11

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1 (1)
(b)	$f'(x) = \frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x}$ Sets $\frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x} = 0 \Rightarrow \ln(4x) = -2$ $Q\left(\frac{1}{4e^2}, -\frac{6}{e}\right)$	M1 A1 dM1 A1 A1 (5)
(c)	Attempts $-2 \times y$ co-ordinate of Q Range $g(x) \leq \frac{12}{e}$	M1 A1ft (2)
		(8 marks)

9 (a)	$(k =) 4\sin^2\left(\frac{\pi}{3}\right) - 1 = 2$ *	B1* (1)
(b)	(i) $x = 4\sin^2 y - 1 \Rightarrow \frac{dx}{dy} = 8\sin y \cos y$ o.e. (ii) Attempts either $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\cos^2 y = \pm 1 \pm \frac{x+1}{4}$ (both for dM1) $\frac{dy}{dx} = \frac{1}{8\sin y \cos y} = \frac{1}{8 \times \sqrt{\frac{x+1}{4}} \times \sqrt{\pm 1 \pm \frac{x+1}{4}}} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	M1 A1 M1 dM1 ddM1 A1* (6)
(c)	At $x = 2$, gradient of curve $= \frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$ Point N (base length of triangle) is solution of $x - \frac{\pi}{3} = -2\sqrt{3}(x - 2) \Rightarrow x = 2 + \frac{\pi}{6\sqrt{3}}$ (= 2.3...) Area $= \frac{1}{2} \times \left(2 + \frac{\pi}{6\sqrt{3}}\right) \times \frac{\pi}{3} = \frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$	M1 dM1 A1 (3)
		(10 marks)

9(a)	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x}{\sin x} + \frac{2 \sin x \cos x}{\cos x}$ (One Correct identity) $= \frac{1 - 2 \sin^2 x}{\sin x} + \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}}$ $= \frac{1}{\sin x} - \frac{2 \sin^2 x}{\sin x} + 2 \sin x = \frac{1}{\sin x} = \operatorname{cosec} x \quad *$	B1 M1 A1* (3)
(b)	E.g. Equation is $\operatorname{cosec}^2 \theta = 6 \cot \theta - 4 \Rightarrow 1 + \cot^2 \theta = 6 \cot \theta - 4$ E.g. $\cot^2 \theta - 6 \cot \theta + 5 = 0$ E.g. $\tan \theta = \frac{1}{5}, 1$ $\theta = 0.197, \frac{\pi}{4}$	M1 A1 dM1 A1, A1 (5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \cot x \, dx = \left[-\operatorname{cosec} x \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ $= 2 - \sqrt{2}$	M1 A1 (2)
		10 marks

6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ $= 2(2x-3)^2 e^{4x-2} \{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	M1 A1 dM1 A1 (4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$ $-27e, g(x), 0g \, g \, \{$	M1, A1 A1 (4)
		(8 marks)

Question Number	Scheme	Marks
6 (a)	$y = (4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} = \right) 2(4x-7)^{-\frac{1}{2}} \quad (\text{see notes})$ At $(8, 5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} \left(= \frac{2}{5} \right)$ Equation for l is $y - 5 = -\frac{5}{2}(x - 8)$ $2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0 *$	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right] \quad (\text{see notes})$ Complete area = $\int_{\frac{7}{4}}^8 (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

10.(a)	$x = 3 \cos 2y \Rightarrow \left(\frac{dx}{dy} = \right) -6 \sin 2y$	M1 A1 (2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2} \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$ $\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	M1 dM1 A1 (3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$ $x^2 = 5 \Rightarrow a = \sqrt{5}$ $\sqrt{5} = 3 \cos 2y \Rightarrow b = \frac{1}{2} \arccos\left(\frac{\sqrt{5}}{3}\right)$	M1 A1 dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6 \sin 2y} = -\frac{1}{4}$ $\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2} \arcsin\left(\frac{2}{3}\right)$ $x = 3 \cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	M1A1 dM1 A1 (4) (9 marks)