8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x}$	M1
	$\dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x} \equiv \dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x} \equiv \dfrac{2\tan x + \tan x - \tan^3 x}{1-\tan^2 x - 2\tan^2 x}$ $\dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$ *	A1*
		(3)

(b)	$\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = 2\sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2\left(1+\tan^2 3\theta\right) - 8$	M1
	$\Rightarrow 2\tan^2 3\theta - \tan 3\theta - 6 = 0$ or e.g. $\Rightarrow 2\tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2\tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\dfrac{3}{2}, 2$ $\tan 3\theta = -\dfrac{3}{2} \Rightarrow 3\theta = \ldots \Rightarrow \theta = \ldots$ or $\tan 3\theta = 2 \Rightarrow 3\theta = \ldots \Rightarrow \theta = \ldots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
		Total 8

Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70°}{1+\tan 2x \tan 70°} = -\dfrac{3}{8} \Rightarrow \tan(2x - 70°) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right)+70°}{2}$	dM1
	awrt 24.7° , awrt 114.7°	A1
		(4)
		(9 marks)

	$\mathrm{f}(x) = 8\sin x\cos x + 4\cos^2 x - 3$	
4(a)	States or uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	M1
	Uses $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $\mathrm{f}(x)$	dM1
	$\left(\mathrm{f}(x) =\right) 8\sin x\cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha = \dfrac{b}{a} \Rightarrow \alpha = ...$ $\left(= \text{"awrt } 0.464\text{"}\right)$	M1
	$\left(\mathrm{f}(x) =\right)$ $2\sqrt{5}\sin\left(2x + 0.464\right) - 1$	A1
		(3)
(c)	(i) Maximum value = $\text{"}2\sqrt{5} - 1\text{"}$	B1 ft
	(ii) Solves $2x + \alpha = \dfrac{5\pi}{2} \Rightarrow x = ...$	M1
	$\left(x =\right)$ awrt 3.69 (or $\left(x =\right)$ awrt 3.70)	A1
		(3)
		(9 marks)

2(a)	$R = 25$	B1
	$\tan\alpha = \dfrac{24}{7} \Rightarrow \alpha = ...$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \dfrac{5}{90 - 3\times\text{"}25\text{"}\times(-1)}$	M1
	$= \dfrac{1}{33}$	A1
(b)(ii)	$\left(2x + \text{"}1.287\text{"}\right) = \pi, ... \Rightarrow x = ...$	M1
	$\Rightarrow x = \dfrac{\pi - \text{"}1.287\text{"}}{2} = 0.927$	A1
		(4)
		Total 7

5(a)	$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$	M1
	$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$	dM1
	$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2(3\sin\theta - 4\sin^3\theta) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6 - 8(1-\cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \dfrac{5-\sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \dfrac{5-\sqrt{41}}{8} \Rightarrow \theta =$ awrt 100° or awrt 260°	A1
	All of 180°, 360°, awrt 100.1°, awrt 259.9°	A1
		(4)
		Total 8

5 (i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
	$x = \dfrac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \dfrac{-5\pm\sqrt{43}}{6} (= -1.926..., 0.2595...) \Rightarrow \theta = \arcsin(...)$	M1
	$\theta = 15.0°, 165°$	A1
		(4) **(7 marks)**

1	$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3(\sec^2\theta - 1) + 7\sec\theta - 3 = 0$	M1
	$3\sec^2\theta + 7\sec\theta - 6 = 0$	A1
	$(3\sec\theta - 2)(\sec\theta + 3) = 0 \Rightarrow \sec\theta = ... \Rightarrow \cos\theta = ...$	dM1
	$\theta = 109.5°, 250.5°$	A1, A1
		(5)

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = ...$	M1
	$x = \text{awrt } 5.8°, \text{ awrt } 84.2°$	A1A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
9(a)	$\dfrac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta} = (2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \dfrac{3}{2}\sin 2\theta = (2+\sec 2\theta)(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)$ **or** $\Rightarrow 3\sin\theta\cos\theta = (2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$	dM1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \dfrac{3}{2}\sin 2\theta = 2\cos 2\theta + 1 \Rightarrow 3\sin 2\theta - 4\cos 2\theta = 2$ *	A1*
		(3)

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **or** $(\alpha =)\ \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	M1
	$\Rightarrow R=\sqrt{3^2+4^2}=...(5)$ **and** $(\alpha =)\ \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right)=...$	**d**M1
	$3\sin 2x-4\cos 2x=2\Rightarrow 5\sin(2x-0.927)=2$ **or** $3\sin 2x-4\cos 2x=2\Rightarrow 5\cos(2x+0.644)=-2$	A1
	$5\sin(2x-0.927)=2\Rightarrow x=\dfrac{\sin^{-1}\frac{2}{5}+0.927}{2}$ **or** $5\cos(2x+0.644)=-2\Rightarrow x=\dfrac{\cos^{-1}\frac{-2}{5}-0.644}{2}$	**dd**M1
	$x=$ awrt 3.81	A1
		(5)
		(8 marks)

7 (a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$ $\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x=\dfrac{1}{2}\cos x+\dfrac{\sqrt{3}}{2}\sin x$	
	$\cos x=\left(\sqrt{3}-2\right)\sin x$	M1
	$\tan x\left(=\dfrac{1}{\sqrt{3}-2}=\dfrac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta=75°,165°$	A1
		(4)

8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1-\cos 2\theta)=\dfrac{2-2\cos 2\theta}{\sin^2 2\theta}$	M1
	$=\dfrac{2-2(1-2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$=\sec^2\theta=1+\tan^2\theta\equiv\text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x-3\sec x-4=0\Rightarrow\sec x=...$	M1
	$\cos x=\dfrac{1}{4}$ (ignore -1)	A1
	$\cos x=\dfrac{1}{4}\Rightarrow x=...$	dM1
	$x=75.5°,\ 284.5°$	A1
		(4)
		(8 marks)