

Question Number	Scheme	Marks
6 (a)	$y = (4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} = \right) 2(4x-7)^{-\frac{1}{2}} \quad (\text{see notes})$ At $(8, 5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} \left(= \frac{2}{5} \right)$ Equation for l is $y - 5 = -\frac{5}{2}(x - 8)$ $2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0 \quad *$	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right] \quad (\text{see notes})$ $\text{Complete area} = \int_{\frac{7}{4}}^8 (4x-7)^{\frac{1}{2}} dx = \left[\frac{(4x-7)^{\frac{3}{2}}}{\frac{3}{2}} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

9(a)	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x}{\sin x} + \frac{2 \sin x \cos x}{\cos x} \quad (\text{One Correct identity})$ $= \frac{1 - 2 \sin^2 x}{\sin x} + \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}}$ $= \frac{1}{\sin x} - \frac{2 \sin^2 x}{\sin x} + 2 \sin x = \frac{1}{\sin x} = \operatorname{cosec} x \quad *$	B1 M1 A1* (3)
(b)	E.g. Equation is $\operatorname{cosec}^2 \theta = 6 \cot \theta - 4 \Rightarrow 1 + \cot^2 \theta = 6 \cot \theta - 4$ E.g. $\cot^2 \theta - 6 \cot \theta + 5 = 0$ E.g. $\tan \theta = \frac{1}{5}, 1$ $\theta = 0.197, \frac{\pi}{4}$	M1 A1 dM1 A1, A1 (5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \cot x \, dx = \left[-\operatorname{cosec} x \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ $= 2 - \sqrt{2}$	M1 A1 (2) 10 marks

6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ $= 2(2x-3)^2 e^{4x-2} \{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	M1 A1 dM1 A1 (4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$ $-27e, g(x), 0g g \text{ ;}$	M1, A1 A1 (4)
(8 marks)		

8. (a)	$f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ $= 2(2x+1)^2 e^{-4x} \{3 - 2(2x+1)\}$ $= 2(2x+1)^2 (1-4x) e^{-4x}$	M1 A1 dM1 A1 (4)
(b)	Sets $f'(x) = 0 \Rightarrow x = -\frac{1}{2}, \frac{1}{4}$ Either $f\left(-\frac{1}{2}\right) = \dots$ or $f\left(\frac{1}{4}\right) = \dots$ Both $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$	B1 M1 A1 (3)
(c)	$\left(\frac{9}{4}, \frac{27}{e}\right)$	B1ft B1ft (2)
9 marks		

10 (a)	$x = \frac{2y^2+6}{3y-3} \Rightarrow \left(\frac{dx}{dy}\right) = \frac{4y(3y-3) - 3(2y^2+6)}{(3y-3)^2}$ $\frac{dx}{dy} = \frac{6y^2 - 12y - 18}{9(y-1)^2} = \frac{2y^2 - 4y - 6}{3(y-1)^2} \text{ o.e}$	M1 A1 dM1, A1 (4)
(b)	P and Q are where $\frac{dx}{dy} = 0$ or where $2y^2 - 4y - 6 = 0$ Solves $2y^2 - 4y - 6 = 0 \Rightarrow 2(y-3)(y+1) = 0 \Rightarrow y = 3, -1$ Subs $y = -1$ and 3 in $x = \frac{2y^2+6}{3y-3} \Rightarrow x = ..$ Achieves $x = -\frac{4}{3}$ and $x = 4$	B1 M1 dM1 A1cso (4)
8 marks		

7(a)	$f(0) = (0-3)^2 = 9$	M1
	$0 \leq f(x) \leq 9$	A1
		(2)
(b)	$f'(x) = -2xe^{-x^2}(2x^2-3)^2 + e^{-x^2} \times 8x(2x^2-3)$	M1A1
	$= 2x(2x^2-3)e^{-x^2}(-(2x^2-3)+4) = 2xe^{-x^2}(2x^2-3)(7-2x^2)$	dM1A1
		(4)
(c)	$x^2 = \frac{3}{2}, \frac{7}{2} \Rightarrow f\left(\sqrt{\frac{7}{2}}\right) = e^{\frac{7}{2}} \left(2 \times \frac{7}{2} - 3\right)^2 = 16e^{\frac{7}{2}}$	M1A1
	$16e^{\frac{7}{2}} < k < 9$	dM1A1
		(4)

5.(a)	$\frac{dy}{dx} = -10 \sin 2x - 24 \cos 2x$	M1, A1
	$\frac{dy}{dx} \Big _{x=\frac{\pi}{3}} = -10 \sin\left(\frac{2\pi}{3}\right) - 24 \cos\left(\frac{2\pi}{3}\right) = -5\sqrt{3} + 12$	A1
		(3)
(b)	$R = 13$	B1
	$\tan \alpha = \frac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	M1A1
		(3)
(c)	Sets $-26 \sin(2x + 1.176) = 6$	M1
	$\sin(2x + "1.176") = -\frac{3}{13}$	A1 ft
	$x = \frac{\arcsin\left(-\frac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	dM1, A1
		(4)
		(10 marks)

9 (a)	$(k =) 4 \sin^2\left(\frac{\pi}{3}\right) - 1 = 2 \quad *$	B1*
		(1)
(b)	(i) $x = 4 \sin^2 y - 1 \Rightarrow \frac{dx}{dy} = 8 \sin y \cos y \quad \text{o.e.}$	M1 A1
	(ii) Attempts either $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\cos^2 y = \pm 1 \pm \frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx} = \frac{1}{8 \sin y \cos y} = \frac{1}{8 \times \sqrt{\frac{x+1}{4}} \times \sqrt{\pm 1 \pm \frac{x+1}{4}}} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}} \quad *$	ddM1 A1*
(c)	At $x = 2$, gradient of curve $= \frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of $\cancel{x} - \frac{\pi}{3} = -2\sqrt{3}(x-2) \Rightarrow x = 2 + \frac{\pi}{6\sqrt{3}} (= 2.3...)$	dM1
	Area $= \frac{1}{2} \times \left(2 + \frac{\pi}{6\sqrt{3}}\right) \times \frac{\pi}{3} = \frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y + 5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)

10.(a)	$x = 3 \cos 2y \Rightarrow \left(\frac{dx}{dy}\right) = -6 \sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1 - kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$	M1
	$\frac{dy}{dx} = -\frac{1}{6\sqrt{1 - \cos^2 2y}} = -\frac{1}{6\sqrt{1 - \left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9 - x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9 - x^2}} = -\frac{1}{4}$	
	$x^2 = 5 \Rightarrow a = \sqrt{5}$	M1 A1
	$\sqrt{5} = 3 \cos 2y \Rightarrow b = \frac{1}{2} \arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6 \sin 2y} = -\frac{1}{4}$	
	$\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2} \arcsin\left(\frac{2}{3}\right)$	M1A1
	$x = 3 \cos 2y = 3\sqrt{1 - \sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	dM1 A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$	M1 A1 dM1, A1 (4)
(b)	Sets $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 4y^2 + 4y - 3 = 0$ $\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \dots$ $P = \left(-1, \frac{1}{2}\right)$ only	M1, A1 dM1 A1 (4) (8 marks)