Question Number	Scheme	Marks
9(a)	$\dfrac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta}=(2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)(\cos\theta-\sin\theta)\left(\cos\theta+\sin\theta\right)$ **or** $\Rightarrow 3\sin\theta\cos\theta=(2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$	**d**M1
	$\Rightarrow \dfrac{3}{2}\sin 2\theta=(2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \dfrac{3}{2}\sin 2\theta=2\cos 2\theta+1\Rightarrow 3\sin 2\theta-4\cos 2\theta=2$ *	A1*
		(3)

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R=\sqrt{3^2+4^2}=\ldots(5)$ **or** $(\alpha=)\ \tan^{-1}\left(\frac{4}{3}\text{ or }\frac{3}{4}\right)=\ldots$	M1
	$\Rightarrow R=\sqrt{3^2+4^2}=\ldots(5)$ **and** $(\alpha=)\ \tan^{-1}\left(\frac{4}{3}\text{ or }\frac{3}{4}\right)=\ldots$	**d**M1
	$3\sin 2x-4\cos 2x=2\Rightarrow 5\sin(2x-0.927)=2$ **or** $3\sin 2x-4\cos 2x=2\Rightarrow 5\cos(2x+0.644)=-2$	A1
	$5\sin(2x-0.927)=2\Rightarrow x=\dfrac{\sin^{-1}\frac{2}{5}+0.927}{2}$ **or** $5\cos(2x+0.644)=-2\Rightarrow x=\dfrac{\cos^{-1}\frac{-2}{5}-0.644}{2}$	**dd**M1
	$x=$ awrt 3.81	A1
		(5)
		(8 marks)

	$f(x) = 8\sin x\cos x + 4\cos^2 x - 3$	
4(a)	States or uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	M1
	Uses $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$	dM1
	$\left(f(x) =\right) 8\sin x\cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha = \frac{b}{a} \Rightarrow \alpha = ...$ $\left(= \text{"awrt } 0.464\text{"}\right)$	M1
	$\left(f(x) =\right)\ 2\sqrt{5}\sin\left(2x + 0.464\right) - 1$	A1
		(3)
(c)	(i) Maximum value = "$2\sqrt{5} - 1$"	B1 ft
	(ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = ...$	M1
	$\left(x =\right)$ awrt 3.69 (or $\left(x =\right)$ awrt 3.70)	A1
		(3)
		(9 marks)

8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1 - \cos 2\theta) = \frac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \frac{2 - 2(1 - 2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$= \sec^2\theta = 1 + \tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = ...$	M1
	$\cos x = \frac{1}{4}$ (ignore -1)	A1
	$\cos x = \frac{1}{4} \Rightarrow x = ...$	dM1
	$x = 75.5°, 284.5°$	A1
		(4)
		(8 marks)

7 (a)	$\sin 4\theta \equiv 2\sin 2\theta\cos 2\theta$	M1
	$\equiv 2(2\sin\theta\cos\theta)(1-2\sin^2\theta)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta(4-8\sin^2\theta)$	A1
		(3)
(b)	$\sec x\sin 4x = 5\sin^3 x\cot x$	
	$\frac{1}{\cos x}\times\cos x\sin x(4-8\sin^2 x) = 5\sin^3 x\cot x$	B1
	$\div\sin x \Rightarrow 4-8\sin^2 x = 5\sin^2 x\cot x$	
	$\div\cos^2 x \Rightarrow 4\sec^2 x - 8\tan^2 x = 5\tan^2 x\cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$ *	A1*
		(3)
(c)	Uses $\sec^2 x = 1+\tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5\pm\sqrt{89}}{8} \Rightarrow x = ...$	dM1
	$\Rightarrow x =$ awrt 0.506, 2.08	A1
		(4)
		(10 marks)

2(a)	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = ...$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90-3\times"25"\times(-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x+"1.287") = \pi, ... \Rightarrow x = ...$	M1
	$\Rightarrow x = \frac{\pi - "1.287"}{2} = 0.927$	A1
		(4)
		Total 7

8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x\cos x+\cos 2x\sin x}{\cos 2x\cos x-\sin 2x\sin x} \equiv \dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$	M1
	$\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x} \equiv \dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x} \equiv \dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x-2\tan^2 x}$ $\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*
		(3)

(b)	$\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = 2\sec^2 3\theta-8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta-8$ $\Rightarrow \tan 3\theta = 2(1+\tan^2 3\theta)-8$	M1
	$\Rightarrow 2\tan^2 3\theta-\tan 3\theta-6=0$ or e.g. $\Rightarrow 2\tan^2 3\theta-\tan 3\theta=6$	A1
	$(2\tan 3\theta+3)(\tan 3\theta-2)=0 \Rightarrow \tan 3\theta=-\dfrac{3}{2},\ 2$ $\tan 3\theta=-\dfrac{3}{2} \Rightarrow 3\theta=\ldots \Rightarrow \theta=\ldots$ or $\tan 3\theta=2 \Rightarrow 3\theta=\ldots \Rightarrow \theta=\ldots$	dM1
	$\theta = 0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

7 (a)	$\sqrt{2}\sin(x+45°) = \cos(x-60°)$	
	$\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°) = \cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x = \dfrac{1}{2}\cos x+\dfrac{\sqrt{3}}{2}\sin x$	
	$\cos x = (\sqrt{3}-2)\sin x$	M1
	$\tan x \left(=\dfrac{1}{\sqrt{3}-2}=\dfrac{\sqrt{3}+2}{-1}\right) = -2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45° = 2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta-45°) = -2-\sqrt{3} \Rightarrow 2\theta-45° = 105°, 285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta = 75°, 165°$	A1
		(4)

5(a)	$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$	M1
	$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$	dM1
	$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2(3\sin\theta - 4\sin^3\theta) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6-8(1-\cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \dfrac{5-\sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \dfrac{5-\sqrt{41}}{8} \Rightarrow \theta =$ awrt 100° or awrt 260°	A1
	All of 180°, 360°, awrt 100.1°, awrt 259.9°	A1
		(4)
		Total 8

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv \dfrac{\cos^4 x - \sin^4 x}{\sin^2 x\cos^2 x}$	M1
	$\equiv \dfrac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x\cos^2 x} \equiv \dfrac{\cos 2x}{\ldots}$ or $\dfrac{\ldots}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \dfrac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\dfrac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x\,\text{cosec}2x$ *	A1*
		(4)
(b)	$4\cot 2\theta\,\text{cosec}2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - \tan^2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - 3\tan^2\theta = 0$	M1
	$\cot^2\theta - 3\tan^2\theta = 0 \Rightarrow \dfrac{1}{\tan^2\theta} - 3\tan^2\theta = 0 \Rightarrow \tan^4\theta = \dfrac{1}{3}$	A1
	$\tan^4\theta = "\frac{1}{3}" \Rightarrow \tan\theta = \pm\sqrt[4]{"\frac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = \ldots$	M1
	$\theta =$ awrt $0.65, \ -0.65$	A1A1
		(5)
		Total 9

Question Number	Scheme	Marks
9(a)	$4\sin\theta\cos\theta = 2\sin 2\theta$	B1
	e.g. $\Rightarrow 6\sin^2\theta\cot 2\theta + 2\sin 2\theta = (3-3\cos 2\theta)\dfrac{\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta$	M1A1
		(3)
(b)	$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$	
	e.g. $\Rightarrow 3\cot 2\theta\sin 2\theta - 14\sin 2\theta = (3-3\cos 2\theta)\cos 2\theta + 2\sin^2 2\theta$	M1
	$\Rightarrow -14\sin 2\theta = -3(1-\sin^2 2\theta) + 2\sin^2 2\theta$	M1
	$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x =)\dfrac{1}{5} \Rightarrow x = ...$	M1
	$x =$ awrt $5.8°$, awrt $84.2°$	A1A1
		(3)
		(9 marks)