8.

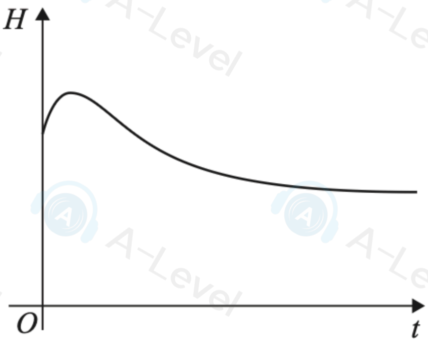

Figure 4

The heart rate of a horse is being monitored.
The heart rate H, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t.

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L.

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T, giving your answer to 3 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1 + 4e^{-0.9t_n}}\right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A.

(1)

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places.

(3)

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

(1)

DO NOT WRITE IN THIS A

9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D\,e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

(a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

(b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

(c) Find the value of T, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRIT

4. The number of bacteria on a surface is being monitored.

The number of bacteria, N, on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

(a) Find the initial number of bacteria on the surface.

(1)

(b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

(c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N, in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000e^{0.1t}}{19 + e^{0.2t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.

(1)

(b) Find $\frac{dN}{dt}$

(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $e^{0.2T} = A$, where A is a constant to be found.

(2)

(d) Hence find the maximum number of squirrels in the forest.

Show your working and give your answer to the nearest whole number.

(3)

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(3)

(d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)