4. $$f(x) = 8\sin x\cos x + 4\cos^2 x - 3$$

(a) Write $f(x)$ in the form

$$a\sin 2x + b\cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R\sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

(3)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) $$f(x) = \sqrt{3}\sin 2x - 3\cos 2x$$

Express $f(x)$ in the form $R\sin(2x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and the exact value of α.

(3)

(b) $$g(x) = \frac{18}{f(3x) + 4\sqrt{3}} \qquad x > 0$$

Using the answer to part (a),

(i) write down the exact minimum value of $g(x)$,

(ii) find the smallest value of x for which this minimum value occurs.

You must make your method clear.

(3)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x-2)\left(\sqrt{3}\sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10\sin\theta = 3\cos 2\theta$$

(4)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$$

show that

$$\tan x = -2 - \sqrt{3}$$

(4)

(b) Hence or otherwise, solve, for $0 \leqslant \theta < 180°$

$$\sqrt{2}\sin(2\theta) = \cos(2\theta - 105°)$$

(4)

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Write $\sin 4\theta$ in the form

$$\sin\theta\cos\theta(P + Q\sin^n\theta)$$

where P, Q and n are constants to be found.

(3)

(b) Use the result from part (a) to show that, for $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

can be written in the form

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

(3)

(c) Use the result from part (b) to solve, for $0 < x < \pi, x \neq \frac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

(4)

8. (a) Prove that

$$2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2\theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3\sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \qquad x \neq (2n+1)\frac{\pi}{6} \qquad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 2\sec^2 3\theta - 8$$

giving your answers to 2 decimal places.

(5)

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Express

$$6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$$

in terms of $\sin 2\theta$ and $\cos 2\theta$ only.

(3)

(b) Hence show that the equation

$$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$$

can be written in the form

$$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$$

(3)

(c) Hence solve, for $0 < x < 90°$, the equation

$$3\cot 2x - 14 = 6\sin^2 x\cot 2x + 4\sin x\cos x$$

giving your answers to one decimal place.

(3)

2: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 7\cos x - 24\sin x$$

(a) Express f(x) in the form $R\cos(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

(b) Using the answer to part (a), find

(i) the minimum value of g(x), giving your answer as a fully simplified fraction,

(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3\operatorname{cosec}\theta = 8\cos\theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180°$

$$\frac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{\cos\theta + \sin\theta} = (2 + \sec 2\theta)(\cos\theta - \sin\theta)$$

can be written in the form

$$3\sin 2\theta - 4\cos 2\theta = 2$$

(3)

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3\sin x\cos x}{\cos x + \sin x} = (2 + \sec 2x)(\cos x - \sin x)$$

giving the answer to 3 significant figures.

(5)