7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that
- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.
(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \qquad t \geqslant 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.
(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)
(3)

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £D, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.
(3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of T, giving your answer to 3 significant figures.
(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.
(2)

8.

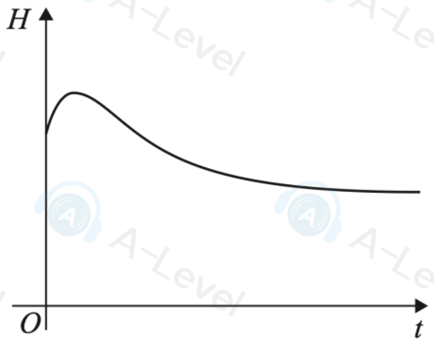

Figure 4

The heart rate of a horse is being monitored.
The heart rate H, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t.

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L.

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T, giving your answer to 3 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1 + 4e^{-0.9t_n}}\right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)