

8. (a) Prove that

$$2 \operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2 \theta \quad (4)$$

(b) Hence solve for $0 < x < 360^\circ$, where $x \neq (90n)^\circ$, $n \in \mathbb{N}$, the equation

$$2 \operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.) (4)

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $0 < \theta \leq 360^\circ$, the equation

$$3 \tan^2 \theta + 7 \sec \theta - 3 = 0$$

giving your answers to one decimal place.

(5)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) $f(x) = \sqrt{3} \sin 2x - 3 \cos 2x$

Express $f(x)$ in the form $R \sin(2x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and the exact value of α .

(3)

(b) $g(x) = \frac{18}{f(3x) + 4\sqrt{3}} \quad x > 0$

Using the answer to part (a),

- write down the exact minimum value of $g(x)$,
- find the smallest value of x for which this minimum value occurs.

You must make your method clear.

(3)

DO NOT WRITE IN THIS AREA

2:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 7 \cos x - 24 \sin x$$

(a) Express $f(x)$ in the form $R \cos(x + \alpha)$ where R and α are constants, $R > 0$

$$\text{and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

(b) Using the answer to part (a), find

(i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,

(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)

5.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)(\sqrt{3} \sec x + 2) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360^\circ$

$$10 \sin \theta = 3 \cos 2\theta$$

(4)

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Write $\sin 4\theta$ in the form

$$\sin \theta \cos \theta (P + Q \sin^n \theta)$$

where P , Q and n are constants to be found.

(3)

(b) Use the result from part (a) to show that, for $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

can be written in the form

$$4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$$

(3)

(c) Use the result from part (b) to solve, for $0 < x < \pi$, $x \neq \frac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

(4)

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Express

$$6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$$

in terms of $\sin 2\theta$ and $\cos 2\theta$ only.

(3)

(b) Hence show that the equation

$$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$$

can be written in the form

$$5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0$$

(3)

(c) Hence solve, for $0 < x < 90^\circ$, the equation

$$3 \cot 2x - 14 = 6 \sin^2 x \cot 2x + 4 \sin x \cos x$$

giving your answers to one decimal place.

(3)

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4. $f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$

(a) Write $f(x)$ in the form

$$a \sin 2x + b \cos 2x + c$$

where a , b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R \sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

(3)

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that $\sin 3x$ can be written in the form

$$P \sin x + Q \sin^3 x$$

where P and Q are constants to be found.

(4)

(b) Hence or otherwise, solve, for $0 < \theta \leq 360^\circ$, the equation

$$2 \sin 3\theta = 5 \sin 2\theta$$

giving your answers, in degrees, to one decimal place as appropriate.

(4)

7.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$$

show that

$$\tan x = -2 - \sqrt{3} \quad (4)$$

(b) Hence or otherwise, solve, for $0 \leq \theta < 180^\circ$

$$\sqrt{2} \sin(2\theta) = \cos(2\theta - 105^\circ) \quad (4)$$

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4 \cot 2x \operatorname{cosec} 2x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta$$

giving your answers to 2 decimal places.

(5)