6:

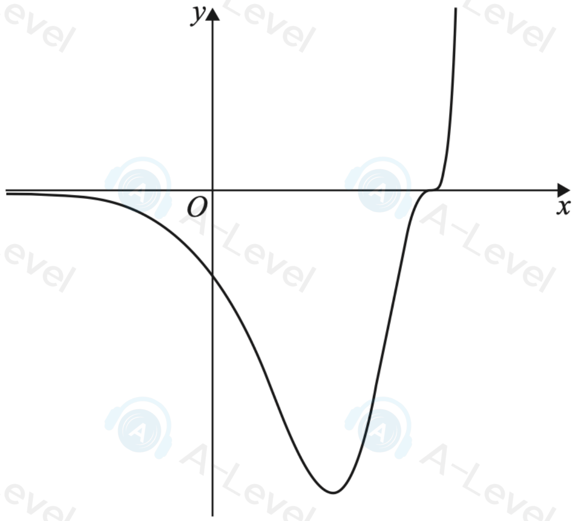

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,

(ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \qquad y \neq -\frac{1}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of y, giving the answer in simplest form.

(4)

A point P lies on C.

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x-axis

(b) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

8.

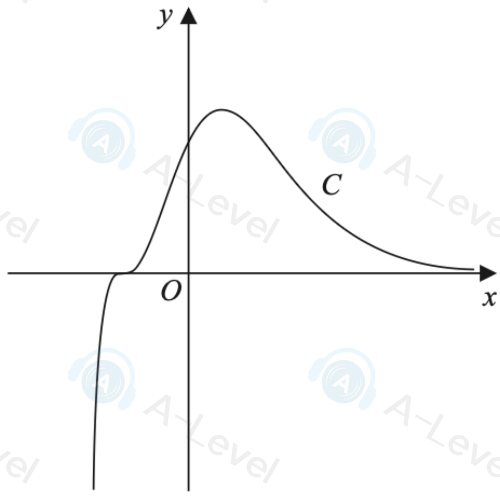

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, \mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\,\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

6. An area of sea floor is being monitored.

The area of the sea floor, S km^2, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$

(2)

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant q

(1)

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

(d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N, in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000e^{0.1t}}{19 + e^{0.2t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.

(1)

(b) Find $\frac{dN}{dt}$

(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $e^{0.2T} = A$, where A is a constant to be found.

(2)

(d) Hence find the maximum number of squirrels in the forest.

Show your working and give your answer to the nearest whole number.

(3)