4.

$$f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$$

(a) Write $f(x)$ in the form

$$a \sin 2x + b \cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R \sin (2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.
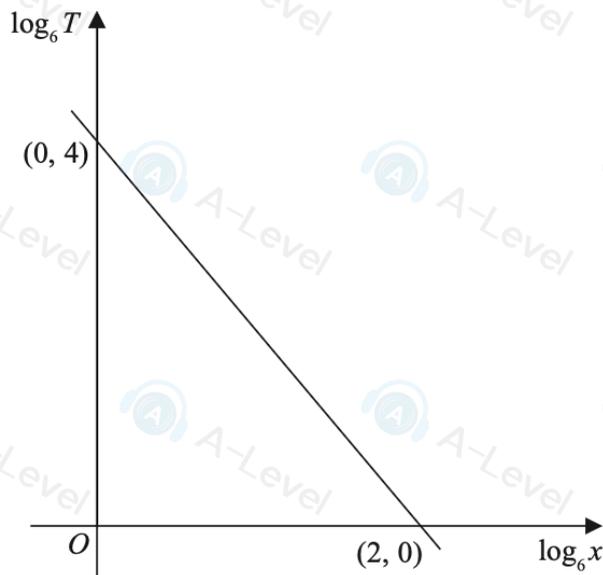
(3)

2.

Figure 1

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points $(0, 4)$ and $(2, 0)$ as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of T when $x = 216$

(3)

(b) Find an equation, not involving logs, linking T with x

(3)

1: The point $P(6, -2)$ lies on the continuous curve with equation $y = f(x)$, $x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 2f(3x)$

(2)

(b) $y = f(x - 2) + 8$

(2)

(c) $y = f^{-1}(x)$

(1)

1: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \frac{2x}{3x + 1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = 4 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Find the value of $gf(1)$

(2)

(b) Find the range of f

(2)

(c) Find $f^{-1}(x)$

(2)

(d) Solve $f^{-1}(x) = f(x)$

(2)

---

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)\left(\sqrt{3} \sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10 \sin \theta = 3 \cos 2\theta$$

(4)

7:   **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Write $\sin 4\theta$ in the form

$$\sin\theta\cos\theta\,(P + Q\sin^n\theta)$$

where P, Q and n are constants to be found.

      (3)

(b) Use the result from part (a) to show that, for $x \neq \dfrac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

can be written in the form

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

      (3)

(c) Use the result from part (b) to solve, for $0 < x < \pi,\ x \neq \dfrac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5\sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

      (4)

---

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £D, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

      where a and b are constants to be found. Give each value to 4 significant figures.

      (3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of T, giving your answer to 3 significant figures.

      (2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

      (2)

3. The share price, £V, of a company is being monitored.

A graph is drawn of $\log_{10} V$ against t, where t is the number of years after monitoring began.

The graph is a straight line passing through the points $(0, 2)$ and $(5, 2.25)$

Using this information,

(a) find an equation for the line in the form

$$\log_{10} V = mt + c$$

where m and c are constants.

(2)

(b) Write the answer to part (a) in the form

$$V = ab^t$$

where a and b are constants to be found.

Give the exact value of a and the value of b to 3 significant figures.

(3)

When $t = T$, the rate of increase in the share price of the company was £50 per year.

(c) Find the value of T, giving your answer to the nearest integer.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

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4. The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

5.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N, in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000e^{0.1t}}{19 + e^{0.2t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.

(1)

(b) Find $\dfrac{dN}{dt}$

(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $e^{0.2T} = A$, where A is a constant to be found.

(2)

(d) Hence find the maximum number of squirrels in the forest.

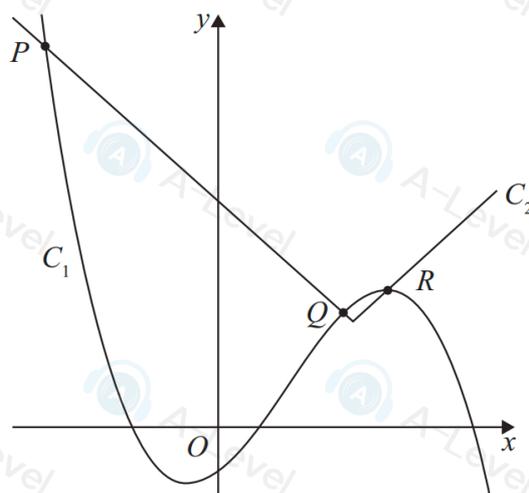Show your working and give your answer to the nearest whole number.

(3)

8.

Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b$$

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2

(d) Find, using algebra, the coordinates of Q. Show each stage of your working.

(Solutions relying on calculator technology are not acceptable.)