3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £D, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of T, giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

(2)

2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6)

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

9.

In this question you must show all stages of your working.

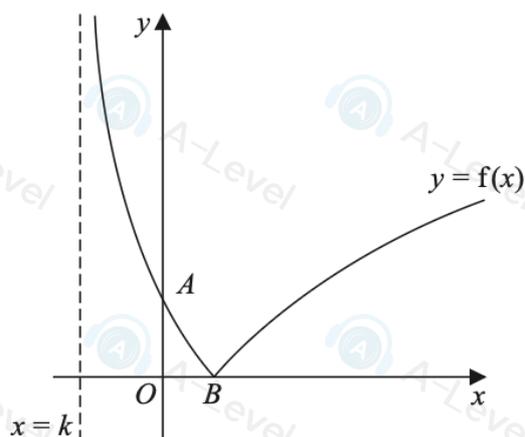**Solutions relying on calculator technology are not acceptable.**

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \left|2 - 4\ln(x + 1)\right| \qquad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$

- cuts the y-axis at point A

- meets the x-axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$\left|2 - 4\ln(x + 1)\right| > 3$$

(5)

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta °C$, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

1: The point $P(6, -2)$ lies on the continuous curve with equation $y = f(x)$, $x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 2f(3x)$

(2)

(b) $y = f(x - 2) + 8$

(2)

(c) $y = f^{-1}(x)$

(1)

2: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 7 \cos x - 24 \sin x$$

(a) Express $f(x)$ in the form $R \cos(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

(b) Using the answer to part (a), find

    (i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,

    (ii) the smallest positive value of x for which this minimum value occurs, giving
    your answer to 3 decimal places.

(4)

---

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \ln(x^2 + 3) \qquad x \in \mathbb{R}$$
$$g(x) = \frac{3 + 5x}{x + 2} \qquad x \in \mathbb{R} \quad x > -2$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $fg(0)$

(2)

(d) Find the exact value of a for which

$$g\left(e^{2a}\right) = f\left(\sqrt{e^4 - 3}\right)$$
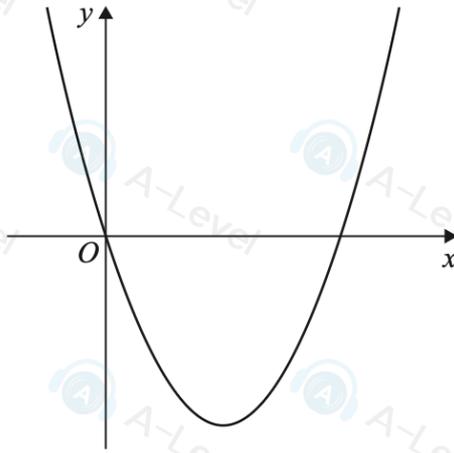
(3)

3.

Figure 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2x^2 - 10x \qquad x \in \mathbb{R}$$

(a) Solve the equation

$$f(|x|) = 48$$

(3)

(b) Find the set of values of x for which

$$|f(x)| \geqslant \frac{5}{2}x$$

(4)

---

8. (a) Prove that

$$2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2 \theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3\sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

1.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $0 < \theta \leqslant 360°$, the equation

$$3 \tan^2 \theta + 7 \sec \theta - 3 = 0$$

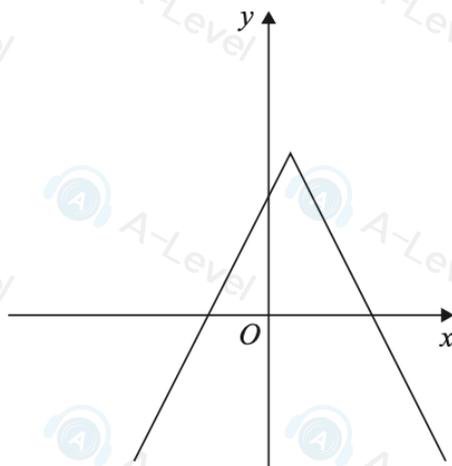giving your answers to one decimal place.

(5)

8.

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b,

   (i) the coordinates of the maximum point of the graph,

  (ii) the coordinates of the point of intersection of the graph with the y-axis,

 (iii) the coordinates of the points of intersection of the graph with the x-axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)