7.

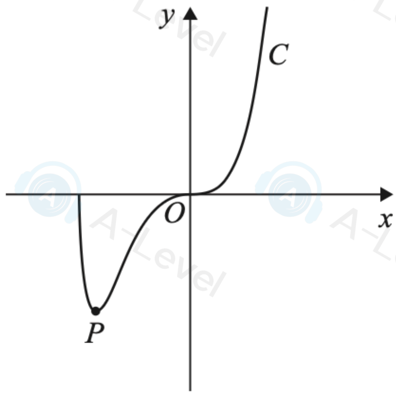

Figure 3

The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3\sqrt{4x+7} \qquad x \geqslant -\frac{7}{4}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{kx^2(2x+3)}{\sqrt{4x+7}}$$

where k is a constant to be found.

(4)

The point P, shown in Figure 3, is the minimum turning point on C.

(b) Find the coordinates of P.

(2)

(c) Hence find the range of the function g defined by

$$\mathrm{g}(x) = -4\,\mathrm{f}(x) \qquad x \geqslant -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C.

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40\,\mathrm{f}\left(x - \frac{3}{2}\right) - 8$$

(2)

9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D\,e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

(a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

(b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

(c) Find the value of T, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that $\sin 3x$ can be written in the form

$$P\sin x + Q\sin^3 x$$

where P and Q are constants to be found.

(4)

(b) Hence or otherwise, solve, for $0 < \theta \leqslant 360°$, the equation

$$2\sin 3\theta = 5\sin 2\theta$$

giving your answers, in degrees, to one decimal place as appropriate.

(4)

8.

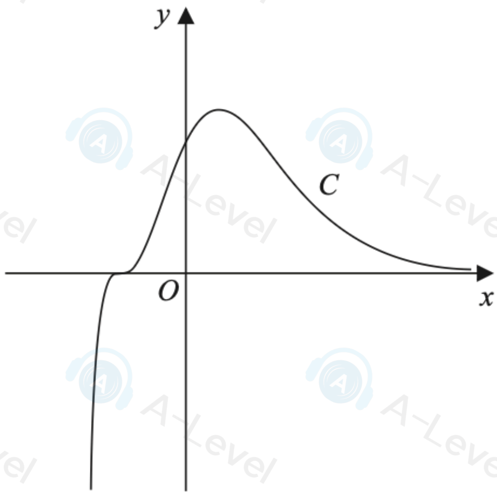

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\,\mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\,\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

---

5. The functions f and g are defined by

$$\mathrm{f}(x) = 2 + 5\ln x \qquad x > 0$$

$$\mathrm{g}(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $\mathrm{f}^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

DO NOT WRITE IN THIS AREA

6:

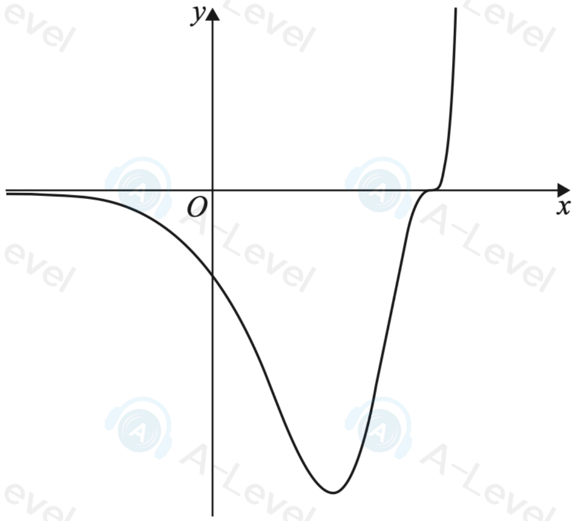

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,

(ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta$$

giving your answers to 2 decimal places.

(5)

DO NOT WRITE IN THIS AREA

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x-2)\left(\sqrt{3}\sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10\sin\theta = 3\cos 2\theta$$

(4)

DO NOT WRITE IN THIS AREA