

2.

$$f(x) = \cos x + 2 \sin x$$

(a) Express $f(x)$ in the form $R \cos(x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $g(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

3. (i) Find $\frac{d}{dx} \ln(\sin^2 3x)$ writing your answer in simplest form.

(2)

(ii)(a) Find $\frac{d}{dx} (3x^2 - 4)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.)

(3)

7. The curve C has equation

$$x = 3 \tan\left(y - \frac{\pi}{6}\right) \quad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C .

Given that the tangent to C at P crosses the x -axis at the point Q .

(b) find, in simplest form, the exact x coordinate of Q .

(5)