4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + q\ln 2$, where p and q are integers.

(5)

DO NOT WRITE IN THIS AREA

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

DO NOT WRITE IN THIS AREA

8. (a) Prove that

$$2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2 \theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3\sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

DO NOT WRITE IN THIS AREA

9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D\,e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

(a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

(b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

(c) Find the value of T, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

2: $$f(x) = 4x^3 + 2x^2 - 12$$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval $[1, 2]$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n + 1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α.

(3)

4. The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

DO NOT WRITE IN THIS AREA

4. $$f(x) = 8\sin x \cos x + 4\cos^2 x - 3$$

(a) Write $f(x)$ in the form

$$a\sin 2x + b\cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $f(x)$ in the form

$$R\sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $f(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WR

5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k.

(1)