

6.

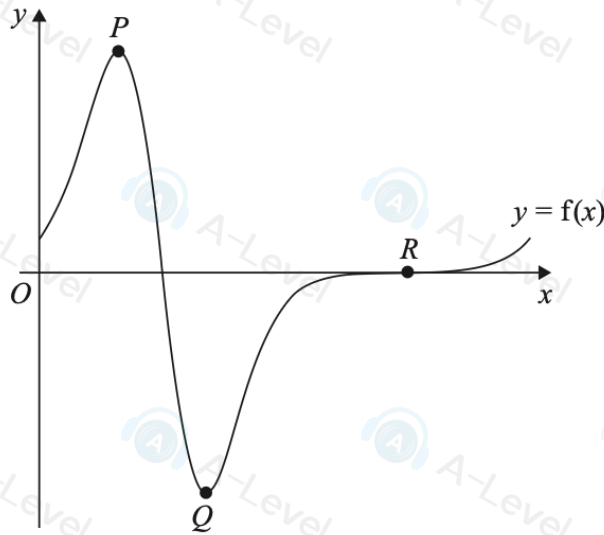


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2e^{3\sin x} \cos x \quad 0 \leq x \leq 2\pi$$

The curve intersects the x -axis at point R , as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q , also shown in Figure 1.

(b) Show that, at points P and Q ,

$$a \sin^2 x + b \sin x + c = 0$$

where a , b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q , giving your answer to 3 decimal places.

(2)

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4 \cot 2x \operatorname{cosec} 2x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta$$

giving your answers to 2 decimal places. (5)

7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \quad 0 \leq x \leq \frac{\pi}{3}$$

The curve has a stationary point at the point P .

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3} \arctan \left(\frac{3}{2x} \right) \quad (4)$$

Using the iteration formula

$$x_{n+1} = \frac{1}{3} \arctan \left(\frac{3}{2x_n} \right) \quad x_1 = 0.4$$

(b) find the value of

(i) x_2 (ii) x_4

giving your answers to 4 decimal places. (3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places. (2)

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} \right)^2 = 6 \cot \theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \right) \cot x \, dx$$

(2)

10.

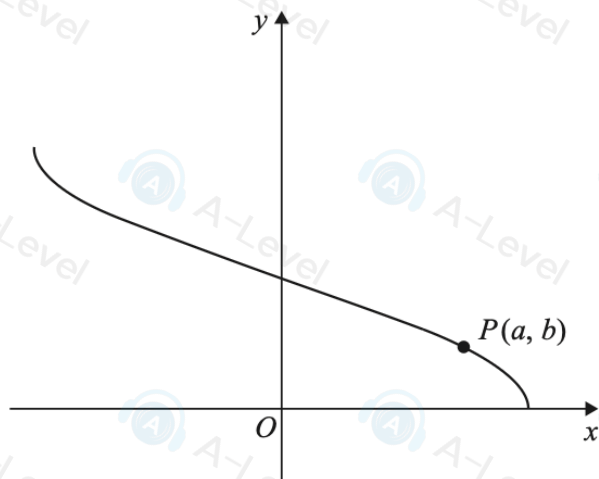


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3 \cos 2y \quad -3 \leq x \leq 3 \quad 0 \leq y \leq \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b .

(4)

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \quad x \in \mathbb{R} \quad x \geq 0$$

$$g(x) = \frac{3}{2x + 1} \quad x \in \mathbb{R} \quad x \geq 0$$

(a) Write down the range of f

(1)

(b) Find the value of $fg(1.5)$

(2)

(c) Find g^{-1}

(3)

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, $\pounds D$, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

- (a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

When $t = T$, the total amount of money raised is $\pounds 45\,000$

According to the model,

- (b) find the value of T , giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of $\pounds 350\,000$ within the first 12 months of monitoring.

According to the model,

- (c) determine whether or not the charity will achieve its aim.

(2)

2:

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 7 \cos x - 24 \sin x$$

- (a) Express $f(x)$ in the form $R \cos(x + \alpha)$ where R and α are constants, $R > 0$

$$\text{and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

- (b) Using the answer to part (a), find

(i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,

(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)