

6. The function f is defined by

$$f(x) = \frac{4x + 3}{x - 2} \quad x \neq 2$$

(a) Find f^{-1}

(3)

(b) Show that

$$ff(x) = \frac{ax + b}{cx + d}$$

where a , b , c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = f(x)$.

(c) Find the point to which P is mapped when $y = f(x)$ is transformed to the curve with equation $y = 2f(3x) + 8$

(2)

7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \quad t \geq 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T , giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

8. (a) Prove that

$$2 \operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2 \theta$$

(4)

(b) Hence solve for $0 < x < 360^\circ$, where $x \neq (90n)^\circ$, $n \in \mathbb{N}$, the equation

$$2 \operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

8.

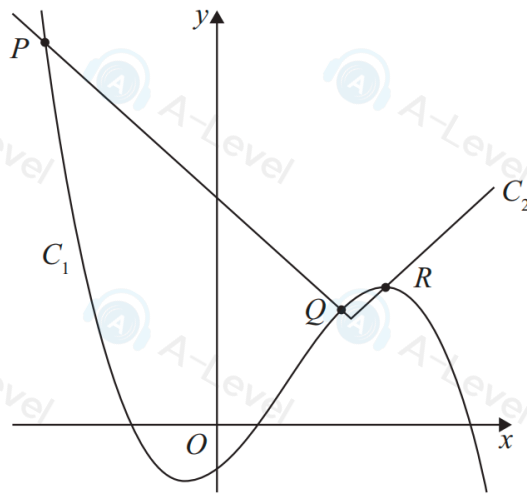


Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

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Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b \quad (2)$$

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b (3)

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2 (2)

(d) Find, using algebra, the coordinates of Q . Show each stage of your working.

(Solutions relying on calculator technology are not acceptable.)

(6)

9.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

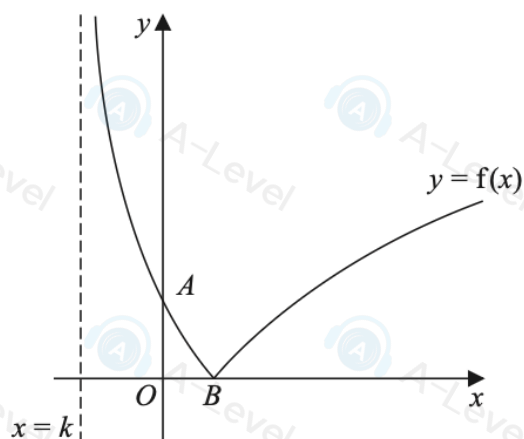


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \quad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y -axis at point A
- meets the x -axis at point B

as shown in Figure 2,

(a) state the value of k (1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B (3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x + 1)| > 3 \quad (5)$$